Original Research

The Water Quality Assessment of Groundwater Based on the TOPSIS-GRA Model

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Abstract

The level of water quality assessment is significant for preventing water pollution; many factors influence its assessment. At first, the TOPSIS-GRA model is introduced; the comprehensive closeness degree of different samples is calculated; finally, the quality level of groundwater is determined based on the comprehensive closeness degree. The conclusions are drawn that results obtained using the suggested method are consistent with the actual investigation for four samples. The accuracy reaches 100%, which is higher than the results from the traditional Matter-Element Extension Model (80%), and estimating the quality level of groundwater using the suggested model is feasible. Since this evaluation method fully utilizes sample data information and combines the advantages of gray correlation analysis and the TOPSIS evaluation model, the evaluation results are more accurate and reasonable than those of a single evaluation method. Therefore, it provides a new method and thoughts to assess the quality level of groundwater in the future.

Keywords: water quality, assessment, groundwater, TOPSIS-GRA model

Introduction

With the rapid development of industry, the issue of groundwater environmental quality has become increasingly prominent [1]. Unlike surface water pollution, groundwater pollution exhibits characteristics such as a slow process, difficulty in detection, and complexity in remediation. Once groundwater is contaminated, even after eliminating pollution sources, it takes a prolonged period for the water quality to recover. Therefore, conducting scientific evaluations of groundwater quality can assist environmental policymakers in understanding the "pollution status quo", providing theoretical references for preventing and controlling groundwater pollution [2].

Currently, there are various methods for evaluating groundwater quality, including the single-index evaluation method [3], the Nemerow pollution index method [4], the fuzzy comprehensive evaluation method [5, 6], the gray clustering method [7], the artificial neural network method [8, 9], the set pair analysis method [10], et al. However, each of these evaluation methods has its limitations to varying degrees. For example, methods such as fuzzy comprehensive evaluation and gray clustering are prone to losing data during the construction of mathematical functions, leading to deviations in the evaluation results. In the evaluation process, various methods involve assigning weights to indicators, and the weighting of each indicator is susceptible to subjective influences. Compared

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with other methods, the TOPSIS-GRA model can address the incompatibility of various indicators through systematic and structural transformations, making it more suitable for groundwater quality assessment [11].

The methods for determining the weights of various indicators in quality evaluation are divided into subjective weighting methods, objective weighting methods, and combined subjective-objective weighting methods [12]. Subjective weighting methods rely excessively on human judgment, which may affect the authenticity and objectivity of the evaluation results. Objective weighting methods use data for mathematical analysis to obtain weights, avoiding human interference. However, they may still fail to take into account the importance of indicators, potentially leading to insufficient applicability [13, 14]. Therefore, combining subjective and objective weighting methods can retain subjective judgments while considering objective realities. As a subjective weighting method, the Analytic Hierarchy Process (AHP) has been widely applied; the CRITIC weighting method is an objective weighting method that comprehensively measures the aim weights of evaluation indicators based on their contrast intensity and the conflicts among them [15]. This method considers both the variability and correlation of indicators; the objective attributes of the data itself are used for evaluation.

The TOPSIS (Technique for Order Preference by Similarity to an Ideal Solution) method is a sorting method based on the proximity of a limited number of evaluation objects to an ideal target; it not only incorporates the thoughts of decision-makers but also fully leverages the information contained in raw data, featuring clear thinking and ease of application. It is suitable for the comprehensive evaluation of complex systems with multiple indicators and multiple schemes and thus has a relatively wide range of applications. For example, Lu Fangyuan et al. [16] comprehensively evaluated water resource carrying capacity in the irrigated areas of desert steppes in Inner Mongolia using the TOPSIS method combined with the entropy weight method; Yan Kun et al. [17] evaluated the water resource carrying capacity in Qinzhou City based on the TOPSIS method; Zhou Xuexin et al. [18] analyzed the water resource carrying capacity of 46 cities in the Pearl River Basin in 2018 using the TOPSIS method; Li Qin et al. [19] evaluated the water resource carrying capacity of the Yangtze River Delta region from 2015 to 2020 using the TOPSIS method.

Based on the results of the former investigation, the TOPSIS-GRA model is applied to evaluate the water quality of groundwater. The TOPSIS-GRA method combines the Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) and Gray Relational Analysis (GRA). Relative to the single method, its evaluation results are more accurate and reasonable. The paper is organized as follows: in second part, methodology based on the TOPSIS-GRA model is presented and in the following part the engineering overview is introduced; in the third part, the assessment

Fig. 1. The location of the survey area.

model of groundwater is constructed, and the assessment results are analyzed; and in the fourth part, conclusions are drawn.

Materials and Methods

Study Area

The research area is located in Zibo City, Shandong Province, with an area of about 20 km^2, involving petrochemicals, chemical manufacturing, and other industries. This is plotted in Fig. 1. The region has a warm, temperate continental monsoon climate with mild weather, distinct seasons, abundant sunshine, moderate precipitation, short frost periods, and extensive evaporation. The annual average temperature is 12.5ºC, and the yearly rainfall is 587 mm. Groundwater in the region is pore water in loose rocks, which is continuously distributed. The shallow pore water is the research object of this investigation. The lithology is mainly silty, fine sand, and silt, with considerable thickness variation and weak water abundance. The primary recharge sources are atmospheric precipitation, surface water infiltration, irrigation return, and underground lateral runoff.

The Combination Weight of the AHP-CRITIC Method

Single weighting can lead to information loss and decision-making bias. In this paper, the subjective and objective weights are determined by the AHP and CRITIC methods, respectively, and then the combined weights are obtained by distance function combination analysis.

The Analysis Hierarchy Method (AHP)

The AHP method is regarded as a subjective static weighting method; its weighting steps are as follows: (1) Determining the hierarchical structure; (2) Establishing the judgment matrix; (3) Consistency test; (4) Determining the index weight. Its calculative process is listed as follows [20]:

Firstly, three hierarchical structures are selected: the target layer (A), the criterion layer (B), and the index layer (C). The three-scale hierarchy method establishes the importance scale comparison matrix between the target layer (A) and the criterion layer (B). The scale of the three-scale theory is shown in Table 1.

$$P = \begin{bmatrix} B_{11} & B_{12} & ... & B_{1m} \\ B_{21} & B_{22} & ... & B_{3m} \\ ... & ... & ... & ... \\ B_{m1} & B_{m2} & ... & B_{mn} \end{bmatrix} \tag{1}$$

Then, the random consistency ratio C_R is calculated using formula (2). When $C_R \leq 0.1$, the consistency of the judgment matrix is acceptable, and the consistency test is passed [21].

$$\begin{cases} C_I = \dfrac{\lambda_{\max} - m}{m - 1} \\ C_R = \dfrac{C_i}{R_i} \end{cases} \tag{2}$$

In the formula: $\lambda_{\max}$ is the largest eigenvalue of the judgment matrix P; m is the order of the judgment matrix P; R_1 is the average random consistency index. The values are shown in Table 2.

Finally, the weighted vector W is obtained by adding the normalized judgment matrix P rows, as shown in Equation (3).

$$\begin{cases} W_L = \dfrac{B_{ij}}{\sum\limits_{i=1}^{n} B_{ij}} \\ W = \dfrac{\sum\limits_{j=1}^{n} W_L}{n} \end{cases} \tag{3}$$

The CRITIC Method

Currently, the commonly used objective weighting methods, such as the entropy weight or coefficient of variation method, have some shortcomings. The more information an attribute contains, the more significant the weight. Considering the volatility of sample data

Table 1. Proportional scale of relative importance for the triple scale approach.

B_{ij} assignment	Implications
1	i is more important than j
0	i is as important as j
-1	j is more important than i

Table 2. R_1 values for 1-10th order judgment matrix.

Order	1	2	3	4	5	6	7	8	9	10
R_1	0	0	0.58	0.9	1.12	1.24	1.32	1.41	1.45	1.49

and the correlation between evaluation indexes, compared with the entropy weight method and coefficient of variation method, the criteria method is more comprehensive. The calculation methods are as follows:

1) The evaluation matrix is established. Assuming that there are m samples and n indexes, the evaluation matrix X can be expressed as:

$$X = \left(x_{ij}\right)_{m \times n} = \begin{bmatrix} x_{11} & x_{12} & ... & x_{1n} \\ x_{21} & x_{22} & ... & x_{2n} \\ ... & ... & ... & ... \\ x_{m1} & x_{m2} & ... & x_{mn} \end{bmatrix} \quad (4)$$

Where, x_{ij} is the j-th assessment index of i-th sample.

2) The evaluation matrix is standardized to obtain the standardized matrix Y. For benefit indicators (the higher the attribute value, the higher the level of water quality) and cost indicators (the higher the attribute value, the lower the level of water quality), the standardization formula is [22]:

$$\begin{cases} y_{ij} = \dfrac{x_{ij} - \min_j\left(x_{ij}\right)}{\max_j\left(x_{ij}\right) - \min_j\left(x_{ij}\right)} (\text{benefit type}) \\ y_{ij} = \dfrac{\max_j\left(x_{ij}\right) - x_{ij}}{\max_j\left(x_{ij}\right) - \min_j\left(x_{ij}\right)} (\text{cost type}) \end{cases} \quad (5)$$

Where, $\max_j(x_{ij})$ and $\min_j(x_{ij})$ are the maximum and minimum magnitudes of the j-th index.

(3) Calculating the mean $\overline{x}_j$ and standard deviation s_j of the score function values for each attribute

$$\overline{x}_j = \frac{1}{m} \sum_{i=1}^{m} x_{ij}, s_j = \sqrt{\frac{1}{m} \sum_{i=1}^{m} \left(x_{ij} - \overline{x}_j\right)^2} \quad (6)$$

(4) Calculating the coefficient of variation V_j of each attribute

$$V_j = s_j \Big/ \overline{x}_j \quad (7)$$

(5) Calculating the correlation coefficient between different indicators. ρ_{kl} is the correlation coefficient between the k-th indicator and the l-th indicator:

$$\rho_{kl} = \frac{\sum_{i=1}^{m} \left(y_{ik} - \overline{y_k}\right)\left(y_{il} - \overline{y_l}\right)}{\sqrt{\sum_{i=1}^{m} \left(y_{ik} - \overline{y_k}\right)^2 \sum_{i=1}^{m} \left(y_{lk} - \overline{y_l}\right)^2}}$$

$$\left(\rho_{kl} = \rho_{lk}; k = 1,2,...,n; l = 1,2,...,n\right) \quad (8)$$

Where, y_{ik} and y_{il} are the standardized values of the k-th and i-th indicators of the i-th evaluation object in the standardized matrix Y, respectively; $\overline{y}_k$ and $\overline{y}_l$ are the average values of the standardized values of the k-th and i-th indicators in the standardized matrix Y, respectively.

(6) Calculating the comprehensive information of the index

$$\mu_j = V_j \sum_{k=1}^{n} \left(1 - \left|\rho_j\right|\right)\left(j = 1,2,...,n\right) \quad (9)$$

(7) Calculating the weights for each attribute:

$$\omega_j = \frac{\mu_j}{\sum_{j=1}^{n} \mu_j} \left(j = 1,2,...,n\right) \quad (10)$$

The Combination Weighting Method

Currently, most combined weighting considers comprehensive weights through the multiplicative combination or linear weighting method. The selection and calculation of the weight preference coefficient are reasonable, but its reliability remains verified. The distance function is introduced to construct the difference relationship equation of subjective and objective weight preference coefficients, and the combined weight is obtained by solving it.

Assuming the combined weight is W_j, the subjective weight vector obtained by the AHP method is α; the objective weight vector obtained by the CRITIC method is β. The weight distribution coefficients of the AHP and CRITIC methods are A and B, respectively. An optimized decision-making model is established as [23, 24]:

$$\begin{cases} \min\left\{\sum_{j=1}^{n}\left[A\left(W_j - \alpha\right)^2 / 2 + B\left(W_j - \beta\right)^2 / 2\right]\right\}, s.t. \\ A + B = 1 \end{cases} \quad (11)$$

By using the Lagrange method, the following formulas are obtained:

$$\begin{cases} W_J = A\alpha_j + B\beta_j \\ A + B = 1 \end{cases} \quad (12)$$

The Euclidean Distance Function L is introduced to solve for the consistency of preference coefficient differences between the Analytic Hierarchy Process (AHP) and the CRITIC Method.

$$\begin{cases} L(\alpha_j,\beta_j) = \sqrt{\sum_{j=1}^{n}(\alpha_j - \beta_j)^2} \\ L(\alpha_j,\beta_j)^2 = (A-B)^2 \end{cases} \tag{13}$$

Where, α_j and β_j are respectively the subjective and objective weight values of the j-th index. By solving the simultaneous Equations (12) and (13), the comprehensive weight W_j can be obtained.

The TOPSIS-GRA Model

1) The calculation steps for improving the traditional TOPSIS method by adopting the gray relational theory and *KL* distance are as follows:

$$R = (r_{ij})_{m\times n} = \begin{bmatrix} r_{11} & r_{12} & ... & r_{1n} \\ r_{21} & r_{22} & ... & r_{2n} \\ ... & ... & ... & ... \\ r_{m1} & r_{m2} & ... & r_{mn} \end{bmatrix} \tag{14}$$

Where R_{ij} is the j-th evaluation index value of the i-th sample.

2) The standardized treatment of evaluation value is performed; the standardization matrix B can be obtained as follows [25]:

$$B = (b_{ij})_{m\times n} = \frac{r_{ij}}{\sqrt{\sum_{i=1}^{m} r_{ij}^2}} (1 \le i \le m, 1 \le j \le n) \tag{15}$$

Where, B_{ij} is the j-th evaluation index value of the i-th sample after standardization treatment.

3) The weighted decision matrix C is obtained by multiplying the normalized matrix B and the combined weights:

$$C = (c_{ij})_{m\times n} = \begin{bmatrix} c_{11} & c_{12} & ... & c_{1n} \\ c_{21} & c_{22} & ... & c_{2n} \\ ... & ... & ... & ... \\ c_{m1} & c_{m2} & ... & c_{mn} \end{bmatrix} = (\omega_j b_{ij})_{m\times n} \tag{16}$$

Where, c_{ij} is the j-th index value of the i-th sample in the weighted decision matrix, W_j is the combined weight value of the j-th evaluation index.

4) The weighted decision matrix C is normalized, and the normalized weighted decision matrix D can be obtained as:

$$D = (d_{ij})_{m\times n} = \frac{c_{ij}}{\sum_{i=1}^{m} c_{ij}} (1 \le i \le m, 1 \le j \le n) \tag{17}$$

Where, d_{ij} is the j-th index value of the i-th sample in the standard weighted decision matrix.

5) The positive ideal solution D^+ and negative ideal solution D^- of the standard weighted decision matrix are determined. For the benefit index set J^+, the positive ideal solution D^+ is the maximum value of the row vector, and the negative ideal solution D^- is the minimum value of the row vector. For the cost index set J^-, the positive and negative ideal solution values are the opposite.

$$\begin{cases} D_j^+ = (d_1^+ \; d_2^+ \; ... \; d_n^+) = \left\{ \left[\max_i(d_{ij}^+)|J\in J^+\right], \left[\min_i(d_{ij}^+)|J\in J^-\right]\right\}, \\ D_j^- = (d_1^- \; d_2^- \; ... \; d_n^-) = \left\{ \left[\min_i(d_{ij}^-)|J\in J^+\right], \left[\max_i(d_{ij}^-)|J\in J^-\right]\right\}, \\ \qquad\qquad\qquad j = 1,2,...,n. \end{cases} \tag{18}$$

Where, D_j^+ and D_j^- are the positive and negative ideal solutions of the j-th evaluation index, respectively.

6) The *KL* distances S_i^+ and S_i^- from the decision object to the positive and negative ideal solutions are calculated, respectively:

$$\begin{cases} S_i^+ = \sum_{j=1}^{n} \left\{ \left| d_j^+ \lg \frac{d_j^+}{d_{ij}} + (1-d_j^+)\lg \frac{1-d_j^+}{1-d_{ij}} \right| \right\}, \\ S_i^- = \sum_{j=1}^{n} \left\{ \left| d_j^- \lg \frac{d_j^-}{d_{ij}} + (1-d_j^-)\lg \frac{1-d_j^-}{1-d_{ij}} \right| \right\}, \end{cases} \tag{19}$$

7) Calculating the gray relational coefficients r_{ij}^+ and r_{ij}^- from the decision-making object to the positive and negative ideal solutions [26]:

$$\begin{cases} r_{ij}^+ = \dfrac{\min\limits_n \min\limits_m |d_j^+ - d_{ij}| + \xi \max\limits_n \max\limits_m |d_j^+ - d_{ij}|}{|d_j^+ - d_{ij}| + \xi \max\limits_n \max\limits_m |d_j^+ - d_{ij}|} \\ r_{ij}^- = \dfrac{\min\limits_n \min\limits_m |d_j^- - d_{ij}| + \xi \max\limits_n \max\limits_m |d_j^- - d_{ij}|}{|d_j^- - d_{ij}| + \xi \max\limits_n \max\limits_m |d_j^- - d_{ij}|} \end{cases} \tag{20}$$

Where, $\min\limits_n \min\limits_m |d_j^- - d_{ij}|$ and $\max\limits_n \max\limits_m |d_j^- - d_{ij}|$ are the maximum and the minimum of the absolute difference between the positive ideal solution of the j-th index and the standard weighted matrix, respectively.

8) The gray relational degrees R_i^+ and R_i^- of the decision-making objects to the positive and negative ideal solutions are calculated as, respectively:

$$\begin{cases} R_i^+ = \dfrac{1}{n}\sum_{j=1}^{n} r_{ij}^+, \\ R_i^- = \dfrac{1}{n}\sum_{j=1}^{n} r_{ij}^- \end{cases} \tag{21}$$

9) Calculating the closeness degree of the decision-making object to the positive and negative ideal solutions:

$$\begin{cases} N_i^+ = L_1 S_i^- + L_2 R_i^+ \\ N_i^- = L_1 S_i^+ + L_2 R_i^- \end{cases} \quad (22)$$

Where, N_i^+ and N_i^- represent the closeness degree of the decision object to the positive ideal solution and the negative ideal solution, respectively, L_1 and L_2 are the coefficients of the decision object for the distance and curve shape, respectively, with $L_1 = L_2 = 5$.

10) Calculating the comprehensive closeness degree F_i^+ of the decision object to the positive ideal solution:

$$F_i^+ = \frac{N_i^+}{N_i^+ + N_i^-} (i = 1, 2, ..., m) \quad (23)$$

The Determination of the Evaluation Index

Considering the indicators' environmental toxicological characteristics and the influence of hydrogeochemical processes in groundwater, 10 indicators from the conventional parameters in the "Quality Standards for Groundwater" (GB/T14848-2017) were selected.

These indicators were classified according to general chemical and toxicological indicators, and a hierarchical model for the groundwater quality system was constructed, as shown in Fig. 2.

The monitoring data of different indicators in the investigation area is shown in Table 3.

The evaluation criteria are shown in Table 4. Level I (very good), level II (good), level III (common), level IV (bad), and level IV (worse).
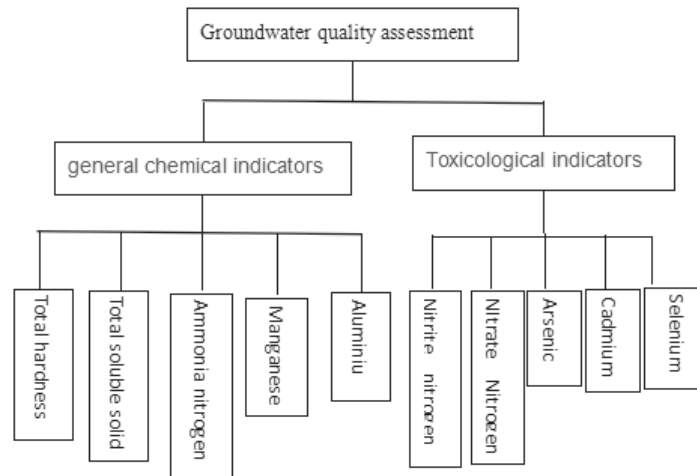

Fig. 2. Hierarchy of groundwater quality evaluation index system.

Table 3. Measured value of water quality evaluation index.

No	Total hardness (mg/L)	TDS (mg/L)	Ammonia Nitrogen (mg/L)	Manganese (µg/L)	Aluminum (µg/L)	Nitrite Nitrogen (µg/L)	Nitrate Nitrogen (µg/L)	Arsenic (µg/L)	Cadmium (µg/L)	Selenium (µg/L)
1#	1260	2510	0.025	866	4.1	0.005	21.2	1.3	0.05	5.99
2#	971	1830	0.025	518	59.2	0.11	142	1.13	0.05	3.92
3#	605	1480	0.025	97.4	2.14	0.005	8	0.92	0.05	1.28
4#	779	1740	0.116	137	16.8	0.005	12.5	1.88	0.17	2.38
5#	2190	5810	0.025	2.87	8.33	0.005	0.0036	45.1	0.05	3.98
6#	111	691	0.025	40.6	3.05	0.005	0.075	1.43	0.59	0.41
7#	883	1830	0.025	8.88	8.22	0.005	10.1	0.87	0.05	1.45
8#	138	308	0.484	154	598	0.005	0.21	5.11	0.05	1.19
9#	4150	7810	0.592	2600	9.86	0.005	0.0535	6.49	0.12	1.1
10#	1720	3070	0.025	538	42.8	0.005	100	7.97	0.07	21.9

Table 4. Standard for evaluation of groundwater quality.

No	Total hardness (mg/L)	TDS (mg/L)	Ammonia Nitrogen (mg/L)	Manganese (μg/L)	Aluminum (μg/L)	Nitrite Nitrogen (μg/L)	Nitrate Nitrogen (μg/L)	Arsenic (μg/L)	Cadmium (μg/L)	Selenium (μg/L)
I	≤150	≤300	≤0.02	≤50	≤10	≤0.01	≤2	≤1	≤0.1	≤10
II	≤300	≤500	≤0.1	≤50	≤50	≤0.1	≤5	≤5	≤1	≤10
III	≤450	≤1000	≤0.5	≤100	≤200	≤1	≤20	≤10	≤5	≤50
IV	≤650	≤2000	≤1.5	≤1500	≤500	≤4.8	≤30	≤50	≤10	≤100
V	≤4150	≤7810	≤2	≤3000	≤598	≤7.2	≤142	≤90	≤15	≤150

The Construction of the Evaluation Frame

The flowchart of the assessment frame is plotted in Fig. 3.

At first, the weights of the predicting index are determined based on a combination of the AHP and the Critical Method. Then, a weighted decision matrix is obtained so the positive and negative ideal solutions are determined; secondly, the approximate degree of the decision object is calculated; finally, the comprehensive closeness is obtained. Based on the closeness, the quality level of groundwater is determined.

Results and Discussion

The Calculation of the Assessment Model

Determination of the Weight Coefficients

(1) Determining the weight coefficient ω_1 based on the AHP method

According to Equations (1)-(3), and in combination with Table 1, the corresponding weight coefficient can be calculated as:

$$\omega_1 = [0.0442\ \ 0.0378\ \ 0.1064\ \ 0.1055\ \ 0.1811\ \ 0.1328\ \ 0.1297\ \ 0.1178\ \ 0.0648\ \ 0.0798]$$
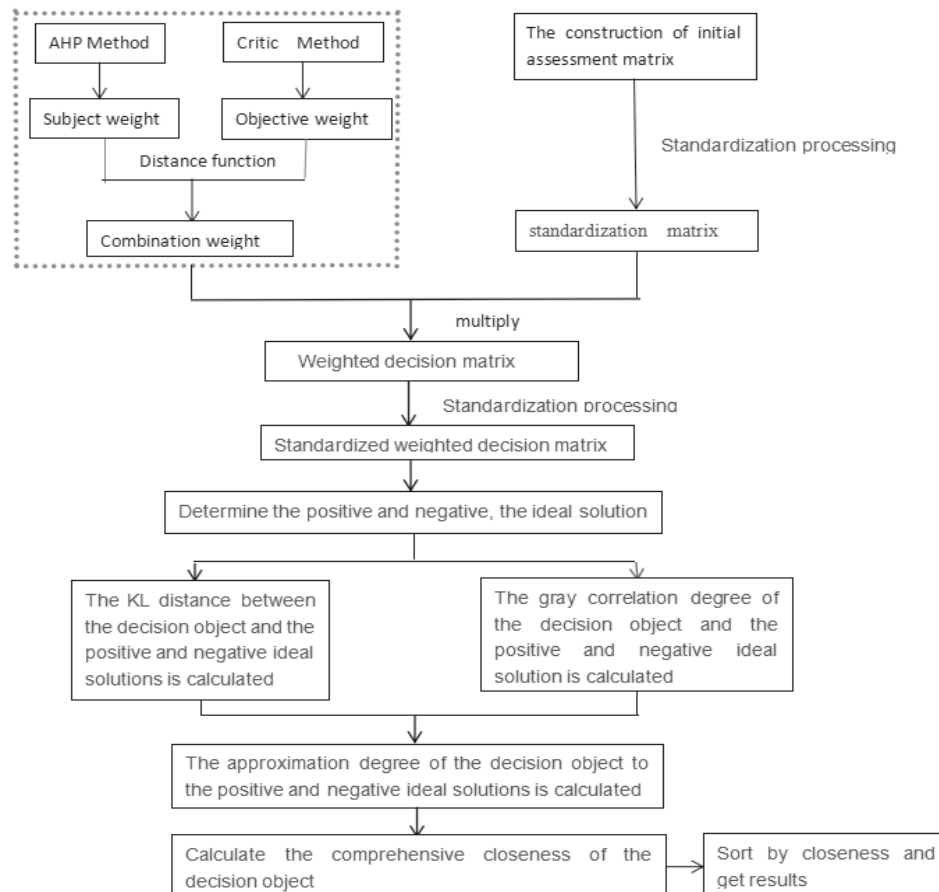

Fig. 3. Flowchart for TOPSIS-GRA model application.

(2) Determining the weight coefficient ω_2 based on the CRITIC method

Based on Equations (8)-(10), and in combination with Table 3, the correlation coefficient can be calculated as:

$$r = \begin{bmatrix} 1 & 0.975 & 0.4565 & 0.8383 & 0.3338 & 0.0909 & 0.0305 & 0.3652 & 0.266 & 0.1535 \\ 0.975 & 1 & 0.375 & 0.7239 & 0.366 & 0.1313 & 0.1177 & 0.5436 & 0.2363 & 0.0919 \\ 0.4565 & 0.375 & 1 & 0.654 & 0.5531 & 0.1826 & 0.3282 & 0.0712 & 0.1019 & 0.2814 \\ 0.8383 & 0.7239 & 0.654 & 1 & 0.1485 & 0.0096 & 0.007 & 0.1202 & 0.1174 & 0.0196 \\ 0.3338 & 0.366 & 0.5531 & 0.1485 & 1 & 0.0305 & 0.1079 & 0.0656 & 0.1831 & 0.1199 \\ 0.0909 & 0.1313 & 0.1826 & 0.0096 & 0.0306 & 1 & 0.795 & 0.1578 & 0.1565 & 0.0242 \\ 0.0305 & 0.1177 & 0.3282 & 0.007 & 0.1079 & 0.795 & 1 & 0.1807 & 0.2456 & 0.5742 \\ 0.3652 & 0.5436 & 0.0712 & 0.1202 & 0.0656 & 0.1578 & 0.1807 & 1 & 0.1851 & 0.0924 \\ 0.266 & 0.2363 & 0.1019 & 0.1174 & 0.1831 & 0.1565 & 0.2456 & 0.1851 & 1 & 0.2318 \\ 0.1535 & 0.0919 & 0.2814 & 0.0196 & 0.1199 & 0.0242 & 0.5742 & 0.0924 & 0.2318 & 1 \end{bmatrix}$$

The standard deviation of different columns is obtained as

$$\mu = [1.6268 \ 1.7028 \ 2.2727 \ 1.9422 \ 2.1976 \ 2.3469 \ 2.3174 \ 2.2127 \ 2.2683 \ 2.2076]$$

Similarly, the weight of each evaluation index can be calculated as:

$$\omega_2 = [0.0771 \ 0.0807 \ 0.1077 \ 0.0921 \ 0.1042 \ 0.1113 \ 0.1099 \ 0.1049 \ 0.1075 \ 0.1047]$$

(3) Calculating the combined weight

Based on Equations (11)-(13), in combination with weight sets ω_1 and ω_2, the combination weight ω can be obtained as follows:

$$\omega = [0.0588 \ 0.0569 \ 0.107 \ 0.0995 \ 0.1469 \ 0.1233 \ 0.1209 \ 0.1121 \ 0.0838 \ 0.0909]$$

The weighting coefficients of the evaluation index are shown in Fig. 4.

The Determination of the Assessment Grade

(1) The determination of the original assessment matrix

Based on Tables 3 and 4, and in combination with Equation (14), the original evaluation matrix can be obtained as:

$$R = \begin{bmatrix} 150 & 300 & 0.02 & 25 & 10 & 0.01 & 2 & 1 & 0.1 & 1 \\ 300 & 500 & 0.1 & 50 & 50 & 0.1 & 5 & 5 & 1 & 10 \\ 450 & 1000 & 0.5 & 100 & 200 & 1 & 20 & 10 & 5 & 20 \\ 650 & 2000 & 1.5 & 1500 & 500 & 4.8 & 30 & 50 & 10 & 50 \\ 1260 & 2510 & 0.025 & 866 & 4.1 & 0.005 & 21.2 & 1.3 & 0.05 & 5.99 \\ 971 & 1830 & 0.025 & 518 & 59.2 & 0.11 & 142 & 1.13 & 0.05 & 3.92 \\ 605 & 1480 & 0.025 & 97.4 & 2.14 & 0.005 & 8 & 0.92 & 0.05 & 1.28 \\ 779 & 1740 & 0.116 & 137 & 16.8 & 0.005 & 12.5 & 1.88 & 0.17 & 2.38 \\ 2190 & 5810 & 0.025 & 2.87 & 8.33 & 0.005 & 0.0036 & 45.1 & 0.05 & 3.98 \\ 111 & 691 & 0.025 & 40.6 & 3.05 & 0.005 & 0.075 & 1.43 & 0.59 & 0.41 \\ 883 & 1830 & 0.025 & 8.88 & 8.22 & 0.005 & 10.1 & 0.87 & 0.05 & 1.45 \\ 138 & 308 & 0.484 & 154 & 598 & 0.005 & 0.21 & 5.11 & 0.05 & 1.19 \\ 4150 & 7810 & 0.592 & 2600 & 9.86 & 0.005 & 0.0535 & 6.49 & 0.12 & 1.1 \\ 1720 & 3070 & 0.025 & 538 & 42.8 & 0.005 & 100 & 7.97 & 0.07 & 21.9 \end{bmatrix}$$
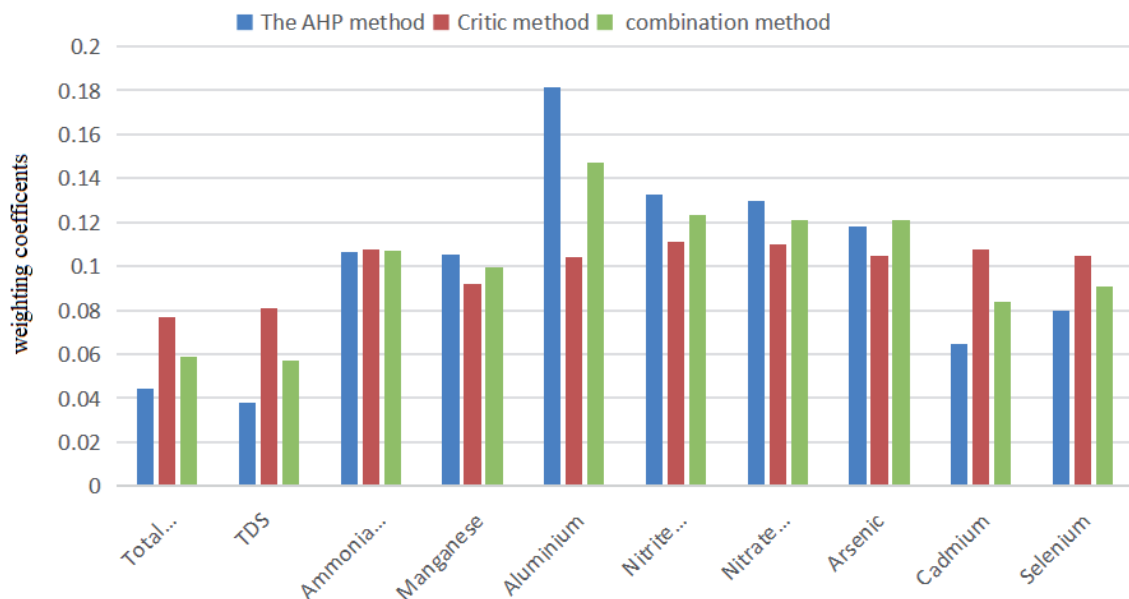

Fig. 4. The weighting coefficients of the evaluation index.

According to Equation (15), the standardized matrix can be obtained as:

$$
B = \begin{bmatrix}
0.0274 & 0.0265 & 0.0113 & 0.0078 & 0.0123 & 0.002 & 0.0111 & 0.0144 & 0.0089 & 0.0168 \\
0.0547 & 0.0441 & 0.0567 & 0.0155 & 0.0617 & 0.0204 & 0.0278 & 0.0722 & 0.0889 & 0.1676 \\
0.0821 & 0.0883 & 0.2834 & 0.031 & 0.2469 & 0.2039 & 0.1113 & 0.1443 & 0.4447 & 0.3352 \\
0.1186 & 0.1766 & 0.8502 & 0.4655 & 0.6173 & 0.9785 & 0.167 & 0.7215 & 0.8894 & 0.838 \\
0.2299 & 0.2216 & 0.0142 & 0.2687 & 0.0051 & 0.001 & 0.118 & 0.0188 & 0.0044 & 0.1004 \\
0.1772 & 0.1616 & 0.0142 & 0.1607 & 0.0731 & 0.0224 & 0.7905 & 0.0163 & 0.0044 & 0.0657 \\
0.1104 & 0.1307 & 0.0142 & 0.0302 & 0.0026 & 0.001 & 0.0445 & 0.0133 & 0.0044 & 0.0215 \\
0.1422 & 0.1536 & 0.0657 & 0.0425 & 0.0207 & 0.001 & 0.0696 & 0.0271 & 0.0151 & 0.0399 \\
0.3996 & 0.5129 & 0.0142 & 0.0009 & 0.0103 & 0.001 & 0 & 0.6508 & 0.0044 & 0.0667 \\
0.0203 & 0.061 & 0.0142 & 0.0126 & 0.0038 & 0.001 & 0.0004 & 0.0206 & 0.0525 & 0.0069 \\
0.1611 & 0.1616 & 0.0142 & 0.0028 & 0.0101 & 0.001 & 0.0562 & 0.0126 & 0.0044 & 0.0243 \\
0.0252 & 0.0272 & 0.2743 & 0.0478 & 0.7383 & 0.001 & 0.0012 & 0.0737 & 0.0044 & 0.0199 \\
0.7573 & 0.6895 & 0.3355 & 0.8068 & 0.0122 & 0.001 & 0.0003 & 0.0937 & 0.0107 & 0.0184 \\
0.3139 & 0.271 & 0.0142 & 0.167 & 0.0528 & 0.001 & 0.5567 & 0.115 & 0.0062 & 0.3671
\end{bmatrix}
$$

Based on Equation (16), the normalized weighted decision matrix can be calculated as:

$$
C = \begin{bmatrix}
0.0016 & 0.0015 & 0.0012 & 0.0008 & 0.0018 & 0.0003 & 0.0013 & 0.0016 & 0.0007 & 0.0015 \\
0.0032 & 0.0025 & 0.0061 & 0.0015 & 0.0091 & 0.0025 & 0.0034 & 0.0081 & 0.0075 & 0.0152 \\
0.0048 & 0.005 & 0.0303 & 0.0031 & 0.0363 & 0.0251 & 0.0135 & 0.0162 & 0.0373 & 0.0305 \\
0.007 & 0.01 & 0.091 & 0.0463 & 0.0907 & 0.1207 & 0.0202 & 0.0809 & 0.0745 & 0.0762 \\
0.0135 & 0.0126 & 0.0015 & 0.0267 & 0.0007 & 0.0001 & 0.0143 & 0.0021 & 0.0004 & 0.0091 \\
0.0104 & 0.0092 & 0.0015 & 0.016 & 0.0107 & 0.0028 & 0.0956 & 0.0018 & 0.0004 & 0.006 \\
0.0065 & 0.0074 & 0.0015 & 0.003 & 0.0004 & 0.0001 & 0.0054 & 0.0015 & 0.0004 & 0.002 \\
0.0084 & 0.0087 & 0.007 & 0.0042 & 0.003 & 0.0001 & 0.0084 & 0.003 & 0.0013 & 0.0036 \\
0.0235 & 0.0292 & 0.0015 & 0.0001 & 0.0015 & 0.0001 & 0 & 0.073 & 0.0004 & 0.0061 \\
0.0012 & 0.0035 & 0.0015 & 0.0013 & 0.006 & 0.0001 & 0.0001 & 0.0023 & 0.0044 & 0.0006 \\
0.0095 & 0.0092 & 0.0015 & 0.0003 & 0.0015 & 0.0001 & 0.0068 & 0.0014 & 0.0004 & 0.0022 \\
0.0015 & 0.0015 & 0.0294 & 0.0048 & 0.1085 & 0.0001 & 0.0001 & 0.0083 & 0.0004 & 0.0018 \\
0.0445 & 0.0392 & 0.0359 & 0.0803 & 0.0018 & 0.0001 & 0 & 0.0105 & 0.0009 & 0.0017 \\
0.0185 & 0.0154 & 0.0015 & 0.0166 & 0.0078 & 0.0001 & 0.0673 & 0.0129 & 0.0005 & 0.0334
\end{bmatrix}
$$

According to Equation (17), the normalized weighted decision matrix can be expressed as:

$$
D = \begin{bmatrix}
0.0104 & 0.0097 & 0.0057 & 0.0038 & 0.0066 & 0.0016 & 0.0057 & 0.0072 & 0.0058 & 0.008 \\
0.0209 & 0.0162 & 0.0287 & 0.0075 & 0.0331 & 0.0165 & 0.0142 & 0.0362 & 0.0576 & 0.0803 \\
0.0313 & 0.0324 & 0.1434 & 0.0151 & 0.1322 & 0.1649 & 0.057 & 0.0724 & 0.2882 & 0.1605 \\
0.0453 & 0.0648 & 0.4302 & 0.226 & 0.3306 & 0.7914 & 0.0854 & 0.3618 & 0.5764 & 0.4013 \\
0.0878 & 0.0813 & 0.0072 & 0.1305 & 0.0027 & 0.0008 & 0.0604 & 0.0094 & 0.0029 & 0.0481 \\
0.0676 & 0.0593 & 0.0072 & 0.078 & 0.0391 & 0.0181 & 0.4044 & 0.0082 & 0.0029 & 0.0315 \\
0.0421 & 0.0479 & 0.0072 & 0.0147 & 0.0014 & 0.0008 & 0.0228 & 0.0067 & 0.0029 & 0.0103 \\
0.0543 & 0.0563 & 0.0333 & 0.0206 & 0.0111 & 0.0008 & 0.0356 & 0.0136 & 0.0098 & 0.0191 \\
0.1525 & 0.1882 & 0.0072 & 0.0004 & 0.0055 & 0.0008 & 0 & 0.3263 & 0.0029 & 0.0116 \\
0.0077 & 0.0224 & 0.0072 & 0.0061 & 0.002 & 0.0008 & 0.0002 & 0.0103 & 0.034 & 0.0033 \\
0.0615 & 0.0593 & 0.0072 & 0.0013 & 0.0054 & 0.0008 & 0.0288 & 0.0063 & 0.0029 & 0.0116 \\
0.0096 & 0.01 & 0.1388 & 0.0232 & 0.3954 & 0.0008 & 0.0006 & 0.037 & 0.0029 & 0.0096 \\
0.2891 & 0.2529 & 0.1698 & 0.3917 & 0.0065 & 0.0008 & 0.0002 & 0.047 & 0.0069 & 0.0088 \\
0.1198 & 0.0994 & 0.0072 & 0.0811 & 0.0283 & 0.0008 & 0.2848 & 0.0577 & 0.004 & 0.1758
\end{bmatrix}
$$

(2) The determination of the gray relational coefficients

According to Equation (18), the positive and negative ideal solutions can be determined as:

$$
D^+ = \begin{bmatrix} 0.2891 & 0.2529 & 0.4302 & 0.3917 & 0.3954 & 0.7914 & 0.4044 & 0.3618 & 0.5764 & 0.4013 \end{bmatrix}
$$

$$
D^- = \begin{bmatrix} 0.0077 & 0.0097 & 0.0057 & 0.0004 & 0.0014 & 0.0008 & 0 & 0.0063 & 0.0029 & 0.0033 \end{bmatrix}
$$

Likely, based on Equations (19) and (20), the gray relational coefficients r_{ij}^+ and r_{ij}^- is obtained as follows:

$$
r_{ij}^+ = \begin{bmatrix}
0.5866 & 0.6191 & 0.4822 & 0.5047 & 0.5042 & 0.3336 & 0.4979 & 0.5272 & 0.4093 & 0.5013 \\
0.5958 & 0.6254 & 0.4961 & 0.5071 & 0.5218 & 0.3378 & 0.5033 & 0.5483 & 0.4325 & 0.5518 \\
0.6053 & 0.6419 & 0.5796 & 0.5121 & 0.6004 & 0.3868 & 0.5322 & 0.5773 & 0.5784 & 0.6215 \\
0.6185 & 0.6775 & 1 & 0.7046 & 0.8592 & 1 & 0.5534 & 1 & 1 & 1 \\
0.6626 & 0.6973 & 0.4831 & 0.6021 & 0.5017 & 0.3333 & 0.5347 & 0.5287 & 0.408 & 0.5281 \\
0.641 & 0.6712 & 0.4831 & 0.5576 & 0.526 & 0.3383 & 1 & 0.5278 & 0.408 & 0.5166 \\
0.6155 & 0.6585 & 0.4831 & 0.5118 & 0.5009 & 0.3333 & 0.5088 & 0.5268 & 0.408 & 0.5027 \\
0.6274 & 0.6679 & 0.499 & 0.5158 & 0.5071 & 0.3333 & 0.5173 & 0.5317 & 0.411 & 0.5084 \\
0.7433 & 0.8592 & 0.4831 & 0.5026 & 0.5035 & 0.3333 & 0.4943 & 0.9177 & 0.408 & 0.517 \\
0.5842 & 0.6316 & 0.4831 & 0.5062 & 0.5012 & 0.3333 & 0.4944 & 0.5294 & 0.4216 & 0.4983 \\
0.6347 & 0.6712 & 0.4831 & 0.5031 & 0.5034 & 0.3333 & 0.5128 & 0.5265 & 0.408 & 0.5036 \\
0.5859 & 0.6194 & 0.5757 & 0.5175 & 1 & 0.3333 & 0.4947 & 0.5489 & 0.408 & 0.5023 \\
1 & 1 & 0.6029 & 1 & 0.5041 & 0.3333 & 0.4944 & 0.5567 & 0.4097 & 0.5018 \\
0.7002 & 0.7203 & 0.4831 & 0.56 & 0.5185 & 0.3333 & 0.7677 & 0.5652 & 0.4085 & 0.6367
\end{bmatrix}
$$

$$r_j^- = \begin{bmatrix} 0.9932 & 1 & 1 & 0.9916 & 0.987 & 0.9979 & 0.9858 & 0.9976 & 0.9928 & 0.9882 \\ 0.9678 & 0.9839 & 0.9451 & 0.9824 & 0.9259 & 0.9619 & 0.9653 & 0.9297 & 0.8783 & 0.837 \\ 0.9436 & 0.9458 & 0.7417 & 0.9643 & 0.7514 & 0.7067 & 0.8741 & 0.8568 & 0.5808 & 0.7154 \\ 0.9133 & 0.8778 & 0.4822 & 0.6367 & 0.5456 & 0.3333 & 0.8223 & 0.5265 & 0.408 & 0.4983 \\ 0.8316 & 0.8467 & 0.9964 & 0.7525 & 0.9967 & 1 & 0.8675 & 0.9922 & 1 & 0.8982 \\ 0.8684 & 0.8886 & 0.9964 & 0.8359 & 0.9129 & 0.958 & 0.4943 & 0.9953 & 1 & 0.9335 \\ 0.9199 & 0.9119 & 0.9964 & 0.9652 & 1 & 1 & 0.9455 & 0.9991 & 1 & 0.9826 \\ 0.8947 & 0.8945 & 0.9349 & 0.9514 & 0.9761 & 1 & 0.9174 & 0.9818 & 0.9828 & 0.9615 \\ 0.7319 & 0.689 & 0.9964 & 1 & 0.9898 & 1 & 1 & 0.5526 & 1 & 0.9324 \\ 1 & 0.969 & 0.9964 & 0.9858 & 0.9985 & 1 & 0.9995 & 0.9899 & 0.927 & 1 \\ 0.8803 & 0.8886 & 0.9964 & 0.9977 & 0.9899 & 1 & 0.9322 & 1 & 1 & 0.9793 \\ 0.9953 & 0.9993 & 0.7482 & 0.9455 & 0.5009 & 1 & 0.9985 & 0.928 & 1 & 0.9844 \\ 0.5842 & 0.6191 & 0.7067 & 0.5026 & 0.9873 & 1 & 0.9996 & 0.9067 & 0.9899 & 0.9862 \\ 0.7791 & 0.815 & 0.9964 & 0.8306 & 0.9363 & 1 & 0.5813 & 0.885 & 0.9971 & 0.6962 \end{bmatrix}$$

(3) The determination of the closeness degree

$$R_i^+ = [0.4966 \; 0.512 \; 0.5635 \; 0.8413 \; 0.528 \; 0.567 \; 0.5049 \; 0.5119 \; 0.5762 \; 0.4983 \; 0.508 \; 0.5586 \; 0.6403 \; 0.5694]$$

$$R_i^- = [0.9934 \; 0.9377 \; 0.8081 \; 0.6044 \; 0.9182 \; 0.8883 \; 0.9721 \; 0.9495 \; 0.8892 \; 0.9866 \; 0.9664 \; 0.91 \; 0.8282 \; 0.8517]$$

Finally, based on Equations (21) and (22), the closeness degree and comprehensive closeness degree can be shown in Table 5.

Its compared results with the actual investigation are plotted. The TOPSIS-GRA model is applied to evaluate the quality level of groundwater. It can be found in Table 5 that the classification standard of groundwater quality is: when $F_i^+<0.0286$, water quality belongs to I; when $0.0286<F_i^+<0.065$, water quality belongs to II;

when $0.065<F_i^+<0.2405$, water quality belongs to III; when $0.2405<F_i^+<0.7783$, water quality belongs to IV; $F_i^+ \geq 0.7783$, it belongs to V.

The quality levels of groundwater from 1# to 10# samples are different. The quality level of groundwater at 2#, 9#, and 10# samples is III; one at 6# sample is I; one at 8# sample is IV; one at the rest samples is II. This means the water quality at 2#, 9#, and 10# samples is common; water quality at 6# sample is very good; water quality at 8# sample is bad; one at the rest samples is good. So, the qualified rate arrives at 90%.

According to the comparative results of the assessment model in Table 6, conclusions demonstrate that the results obtained by the suggested method are entirely consistent with the actual investigation for

Table 6. The comparison results.

Sample number	Text method	Matter-Element Extension Model	Current specifications
1#	II	II	II
2#	III	II	III
3#	II	II	II
4#	II	II	II
5#	II	II	II
6#	I	I	I
7#	II	II	II
8#	IV	IV	IV
9#	III	II	III
10#	III	III	III

Table 5. Results of the comprehensive closeness degree.

Samples	N_i^+	N_i^-	F_i^+	Level
I-II level	0.2555	8.6901	0.0286	-
II-III level	0.3654	5.2540	0.0650	-
III-IV level	0.8160	2.5773	0.2405	-
IV-V level	2.7356	0.7794	0.7783	-
1#	0.4417	7.2270	0.0576	II
2#	0.6560	5.4741	0.1070	III
3#	0.2948	8.4592	0.0337	II
4#	0.3354	6.9206	0.0462	II
5#	0.6315	9.3306	0.0634	II
6#	0.2667	9.2649	0.0280	I
7#	0.3082	8.5268	0.0349	II
8#	0.6105	7.6115	0.0743	IV
9#	0.9398	7.1125	0.1167	III
10#	0.6969	5.9059	0.1055	III

ten different samples. The accuracy reaches 100% for the proposed method, which is higher than the results from the Matter-Element Extension Model (80%). So, the conclusion demonstrates that estimating groundwater quality using the suggested model is feasible. The method can provide more details for assessing groundwater quality; for example, the aluminum of the 3# sample is 2.14, which should belong to level I, as shown in Table 3. In addition, the quality of the other indicators obtained using the suggested model belongs to level II, so the quality level probability of the 3# sample at level II is higher than that of levels I, III, IV, and V. So, the quality of the 3# sample only belongs to level II and almost impossibly belongs to levels I, III, IV, and V. Furthermore, the quality level of the 1# sample is more likely to belong to level II than that of the 3# sample because its comprehensive closeness degree (0.0567) for grade II is higher than that of the 1# sample (0.0337). The results obtained using the suggested model accurately demonstrate the groundwater quality and further determine the intensity grade ranking for different samples at the same level.

Conclusions

Considering Total Hardness, TDS, Ammonia Nitrogen, Manganese, Aluminum, Nitrite Nitrogen, Nitrate Nitrogen, Arsenic, and Cadmium, as well as Selenium, a new evaluation method is introduced in this manuscript to assess the quality grade of groundwater based on the TOPSIS-GRA model. First, the 10 different assessment indicators are determined. Then, the weight coefficients are calculated according to the AHP-CRITIC method. Finally, the comprehensive closeness degree of different samples is calculated using the TOPSIS-GRA model, and the quality level of groundwater is determined according to the comprehensive closeness degree.

The TOPSIS-GRA model is applied to evaluate the quality grade of groundwater. Conclusions demonstrate that the water quality at 2#, 9#, and 10# samples is common; water quality at 6# sample is very good; water quality at 8# sample is bad; one at the rest samples is good. So, the qualified rate arrives at 90%. The results obtained by the suggested method are entirely consistent with the actual investigation for ten different samples. Its accuracy reaches 100% for the proposed method, which is higher than the results from the Matter-Element Extension Model (80%). Besides, the result demonstrates estimating the groundwater quality using the suggested model is feasible.

In total, this evaluation method fully utilizes the information from sample data and combines the advantages of gray correlation analysis and the TOPSIS evaluation model. The evaluation results are more accurate and reasonable than those of a single evaluation method. The results obtained using the suggested model accurately demonstrate the groundwater quality and

further determine the level grade ranking for different samples at the same level.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

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