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Original Research

The Application of the Game-Gray Target Model on the Water Quality Assessment in the Tao Jiang River

Li-Ping Guo[1], Xin-Chao Wu[2], Xin-Bao Gu[3]*

[1]School of Architecture, Nanyang Institute of Technology, Nanyang, Henan, China
[2]Shanghai Bounce and Jump Technology Group Co. , Ltd. Shanghai, China
[3]School of Civil Engineering, Nanyang Institute of Technology, Nanyang, Henan, China

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Abstract

Water quality evaluation is of great significance in protecting the water environment. Dissolved oxygen (X_1), Permanganate index (X_2), BOD_5 (X_3), Ammonia nitrogen (X_4), Total phosphorus (X_5), and Fluoride (X_6) are selected as the assessment index, and the game-gray target model is introduced. Then, the gray target water quality model in the Tao Jiang River section is established. The weight coefficients of each index are calculated using the game combination weighting method, and each sample's synthetic target center distance is determined using the gray target model. Finally, the levels of water quality are determined. The suggested model can mine the small sample data to the greatest extent and weaken the information shortage problem caused by the small sample to a certain extent. The suggested model is feasible, has higher validity and accuracy in evaluating water quality, and has better practical application prospects. The final quality grade of water can be evaluated quantitatively. Therefore, it can provide a new approach and a new way to think about future water quality assessments.

Keywords: assessment, water quality, the Tao River, game-gray target model

Introduction

Water quality refers to water's chemical, physical, and biological characteristics; it embodies water quality and can reflect the degree of water cleanliness [1]. It is also an essential criterion for judging the function of water. The quality of water in the environment is vital to the life and survival of all species that depend on it; the evaluation of water quality is the premise of protecting the water environment and treating water pollution. Because the water environment and human water use functions are different, the evaluation standard and method are also different. So, selecting the most suitable evaluation method for water quality can point in the direction of understanding water pollution and protecting the water environment [2, 3].

For the evaluation of water quality, many scholars have investigated and analyzed it using different methods [4]. As early as the 1960s, the concept of the water quality index (QI) was proposed by American scholars [5], and more and more water quality assessment methods have been proposed since then. The most

*e-mail: 15823405952@163.com

representative method is the water quality index (WQI), which was put forward by Brown et al. in 1970 [6]. It quantifies the water quality by calculating the single and comprehensive WQI and realizes the division of water quality [7]. At present, the common methods of water quality evaluation at home and abroad are the single-factor pollution index method [8] and the comprehensive pollution index method [9]. These two methods are essential for national surface water quality assessment. In addition, the Nemerow index method [10] and the principal component analysis method [11]. In recent years, the rapid development of computer technology has not only improved the efficiency of data processing but also derived a series of more scientific methods of water quality data processing. For example, the fuzzy mathematics method [12], the gray index method [13], the analytic hierarchy process [14], the artificial neural network method [15], and the overall level of water quality evaluation have a qualitative leap.

The above models and methods improved the development of pavement quality assessment. However, they still have some shortcomings [16]. For example, the training process for the neural network model needs a lot of labeled sample data, and feature selection will significantly affect the effect of the final model. Principal component analysis (PCA) omits some essential information from small data, which is vital for data distribution [17]. Besides, the single-factor pollution index method needs to be revised to reflect the comprehensive state of the water body objectively. The above shortcomings greatly restrict the development of water quality evaluation.

To overcome the shortcomings of the abovementioned methods, the game-gray target model is applied to assess the level of water quality. Gray target decision-making is a kind of uncertainty system that studies the information of just a few samples and poor information [18]. It can mine and develop the data to the greatest extent based on the known information from small samples. The theory has been applied to finance, the military, and other fields and has achieved good results [19]. It will be used to evaluate the water quality level in the paper, and the procedure will be analyzed.

The paper is organized as follows: Section 2 introduces the theory and methodology based on the game-gray target model. Section 3 provides an engineering example of water quality assessment. Section 4 establishes the assessment model and analyzes the results. Section 5 provides the conclusions.

Materials and Methods

Study Area

The Tao Jiang River is the primary tributary and main source of the Gan River, in Jiangxi Province, China. Its location is plotted in Fig. 1. It originates from the northeast slope of the Jiulian mountain range at the junction of Guangdong and Jiangxi and flows through Guangdong and Jiangxi provinces. Its main tributaries include the Thai Binh River, the Ottawa River, and the Huangtian River, which eventually joined the Gongjiang River in Gan County. The river basin is located

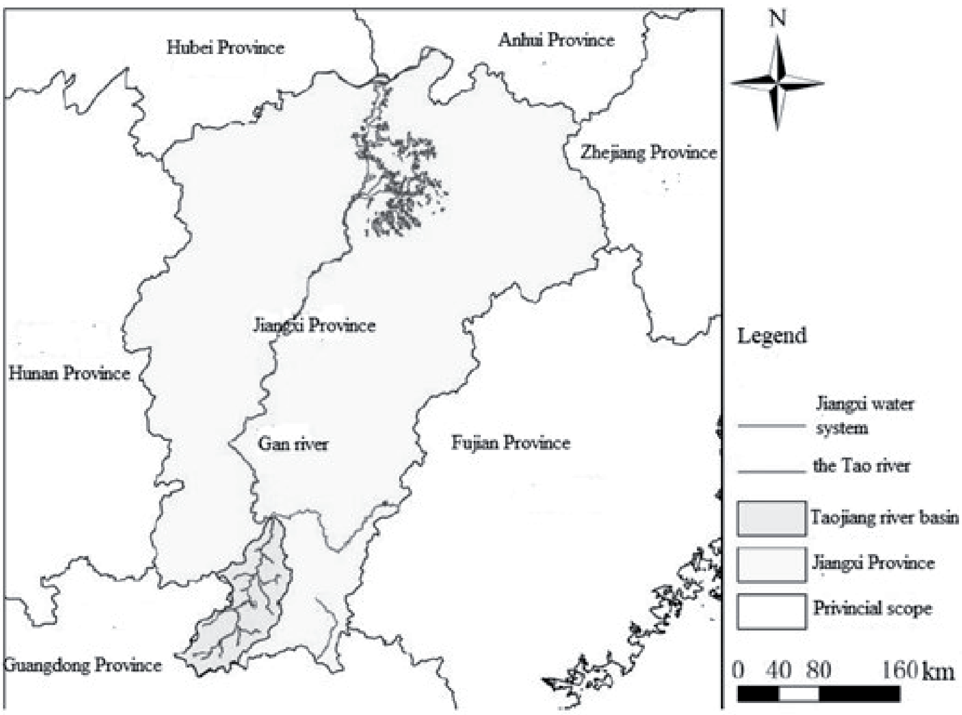

Fig. 1. The location of the study area.

in the southwest of Ganzhou, Jiangxi province (24°28′-25°53′N, 114°11′-115°19′ E). It covers the whole of Nan County, Longnan City, most of Xinfeng County, Gan County District, Anyuan County, and parts of Dingnan County. The basin area is approximately 8,440 km^2. The annual average precipitation is 2300mm, and the difference in rainfall in a year is noticeable; it mainly focuses between April and August. The interannual change is also apparent; the high and low-water years appear alternately.

The Determination of the Assessment Index

In this paper, six indicators are selected as the evaluation indexes of water quality based on the relevant norms and the research results of predecessors; they are, respectively, Dissolved oxygen (X_1), Permanganate index (X_2), BOD_5 (X_3, Ammonia nitrogen (X_4), Total phosphorus (X_5) and Fluoride (X_6). The evaluation indexes were divided into six quality evaluation grades: level I (excellent), level II (good), level III (common), level IV (inferior), level V (bad), and level VI (worse). Twelve groups of monitoring data and assessment results in the Tao Jiang River basin are selected as learning samples; the specific data is shown in Table 1.

The Game-Gray Target Model

The gray target decision-making approach is an essential method for solving multi-attribute decision-making problems from an objective angle, and it can effectively reduce the loss of original data information in the decision-making process [20]. Its basic idea is that the optimal data sequence is found to construct a standard model from the existing set of sequences. Then, the standard model is applied as the target, the gray target model is built by comparing other models with the standard one, the approaching degree between the models is evaluated, and the target is judged; finally, the target distance is calculated, and the evaluation grade is determined. Considering the accuracy defect of the single-weight gray target model, the game combination weighting method is adopted in the paper, the combination weight of the critic and entropy methods is optimized, and the optimal weight is obtained.

(1) The establishment of a decisive matrix

It is assumed that there are m samples to be evaluated $A_i(i = 1, 2, ..., m)$, n evaluation indexes $A_j(j = 1, 2, ..., m)$, and then the sample matrix is $A = \{a_{ij}\}(i = 1, 2, ..., n)$.

Suppose that c_j is the mean value of the evaluation index, then:

$$c_j = \frac{1}{m}\sum_{i=1}^{n} a_{ij} \tag{1}$$

Where, $i = 1, 2, ..., m; j = 1, 2, ..., n$.

For x_{ij} standardized processing results, the economic indicators can be expressed as:

$$x_{ij} = \frac{a_{ij} - c_j}{\max\left(\max\{a_{ij}\} - c_j, c_j - \min\{a_{ij}\}\right)} \tag{2}$$

For a cost-type indicator, the formula is

$$x_{ij} = \frac{c_j - a_{ij}}{\max\left(\max\{a_{ij}\} - c_j, c_j - \min\{a_{ij}\}\right)} \tag{3}$$

According to Equations (1)-(3), the decisive matrix can be expressed as:

Table 1. The learning sample of water quality.

Sample sequence	X_1	X_2	X_3	X_4	X_5	X_5	Assessment grade
1	10.35	1.29	1.27	0.13	0.01	0.63	I
2	10.67	0.76	0.28	0.08	0	0.36	I
3	6.76	3.34	0.59	0.41	0.02	0.82	II
4	6.3	2.56	1.1	0.45	0.08	0.84	II
5	5.49	4.76	3.25	0.86	0.19	0.63	III
6	5.81	5.9	3.73	0.81	0.16	0.99	III
7	3.25	8.98	4.95	1.22	0.22	1.22	IV
8	3.96	7.01	4.17	1.46	0.3	1.22	IV
9	2.75	13.04	6.93	1.77	0.35	1.17	V
10	2.17	13.24	6.01	1.7	0.35	1.47	V
11	0.42	15.45	10.63	3.96	0.48	2	VI
12	0.76	15.3	10.41	6.97	0.46	1.88	VI

$$X = \begin{Bmatrix} x_{11} & x_{12} & \dots & x_{1n} \\ x_{21} & x_{22} & \dots & x_{2n} \\ \dots & \dots & \dots & \dots \\ x_{m1} & x_{m2} & \dots & x_{mn} \end{Bmatrix}$$

(2) The calculation of the target center distance

For the decisive matrix X, if $x_j^{0+} = \max\{x_{ij}|1<i<m\}$, then $x^{0+} = \{x_1^{0+}, x_2^{0+}, \dots, x_m^{0+}\}$ is called the positive target center of the gray target; $x^{0-} = \{x_1^{0-}, x_2^{0-}, \dots, x_m^{0-}\}$ is defined as the negative target center. The distance between x^{0+} and x^{0-} is regarded as the interval d^0 between the positive and negative target centers, and

$$d^0 = \left|x^{0+} - x^{0-}\right| = \left(\sum_{j=1}^{n} \omega_j \left(x_j^{0+} - x_j^{0-}\right)^2\right)^{1/2} \tag{4}$$

Where, ω_i is the optimal weight of the *jth* index obtained based on the game theory.

The positive target center distance d_i^+ is the distance between x_i and x^{0+}, its formula is:

$$d_i^+ = \left|x_i - x^{0+}\right| = \left(\sum_{j=1}^{n} \omega_j \left(x_j - x_j^{0+}\right)^2\right)^{1/2} \tag{5}$$

The negative target center distance d_i^- is the distance between x_i and x^{0-}, its formula is:

$$d_i^- = \left|x_i - x^{0-}\right| = \left(\sum_{j=1}^{n} \omega_j \left(x_j - x_j^{0-}\right)^2\right)^{1/2} \tag{6}$$

The distance from any sample point x_i to the positive target center is $d_i^+<d_0$, then x_i is located in the sphere with x^{0+} as the center and d^0 as the radius; the distance from the sample point x_i to the negative target center is $d_i^-<d_0$, then x_i is located in the sphere with x^{0-} as the center and d^0 as the radius. Any sample point x_i, the positive target center d_i^+, and the negative target center d_i^- are three points in space. The three points are collinear or triangular. Therefore, the dangerous degree of the samples can be measured using the projection size of the positive target center distance on the line between the positive and negative target centers. Assuming the angle between the positive target center distance and the line between the positive and negative target centers is θ, then according to the law of cosines, it can be obtained:

$$d_i^* = d_i^+ \cos\theta = \frac{(d_i^+)^2 + (d^0)^2 - (d_i^-)^2}{2d^0} \tag{7}$$

(3) The classification of quality grade

From the definition of target center distance, it can be found that the comprehensive target distance can quantitatively reflect the quality grade of samples. Assuming that samples to be evaluated have t quality grades, let $D = (d_1, d_2, \dots, d_m)$ be comprehensive target center distance set S of samples to be evaluated, the ordered set of t quality grades is $B = (B_1, B_2, \dots, B_t)$, let f is a positive integer, and $1\leq f<t$, then the threshold of the *fth* set $g_f = \max\{B\}$, $c_f = \max\{B_f\}$, then $h_f = \alpha c_f + (1-\alpha)g_f$, and $\alpha\in(0,1)$, $h_0 = 0$, $h_q = +\infty$, $d = (d_1, d_2, \dots, d_i)$ are the critical comprehensive target center distance of different quality grades, the interval distribution set of comprehensive target distance for t group quality grades can be obtained as follows:

$$D_{ij} = \left\{d \middle| h_0 > d_1 > h_1, \dots, h_{t-1} > d_t > h_t\right\} \tag{8}$$

The Determination of Index Weights

(1) The critical method

The critic method is a kind of weight-assignment method [21], which makes use of the variability and conflict of different evaluation indexes to determine weight, and it can comprehensively measure the evaluation index. Its procedure is listed as follows:

① The standardization and processing of data

Each evaluation index is quantified dimensionlessly to eliminate the influence of different variables. If the evaluation index is a benefit type, the calculation formula is:

$$y = \frac{x_{ij} - \min(x_{ij})}{\max(x_{ij}) - \min(x_{ij})} \tag{9}$$

If the evaluation index is a cost type, the calculation formula is

$$y = \frac{\max(x_{ij}) - x_{ij}}{\max(x_{ij}) - \min(x_{ij})} \tag{10}$$

Where y is a normalized processing value. $\max(x_{ij})$ and $\min(x_{ij})$ are the maximum and minimum of a set of evaluation indicators.

② The variability of evaluation indexes is usually expressed using the standard deviation σ_j of the evaluation index, and the calculation formula is

$$\sigma_j = \sqrt{\frac{\sum_{i=1}^{n}\left(x_{ij} - x_j\right)^2}{n}} \tag{11}$$

Where x_j is the mean of *jth* evaluation index; n is the total number of the *jth* evaluation index.

③ The correlation coefficient of the evaluation index was calculated as:

$$r_{xy} = \frac{\sum\left(x-\bar{x}\right)\left(y-\bar{y}\right)}{\sqrt{\sum\left(x-\bar{x}\right)^2\sum\left(y-\bar{y}\right)^2}} \tag{12}$$

④ The conflict coefficient of the evaluation index was calculated as:

$$C_j = \sum_{m=1}^{n}\left(1-r_{mj}\right) \tag{13}$$

⑤ Calculating the weight coefficient ω_j of the evaluation index:

$$\omega_j = \frac{\sigma_j \times C_j}{\sum_{j=1}^{n}\sigma_j \times C_j} \tag{14}$$

(2) The entropy method

The entropy weight method is a method to determine the entropy value of an index according to the variance size of the evaluation index; its specific calculation method is shown in the literature [22].

(3) The game theory combination weighting method

In order to avoid the information loss caused by a single weighting method and improve the accuracy of the weights, the combination weighting method of game [23, 24] is applied to optimize the weights obtained by several weighting methods, and the consistency is found out, and the optimal weight is obtained.

① The weight sets ω_1 and ω_2 are obtained by the entropy weight and CRITIC methods. It is assumed that α_1 and α_2 are the linear combination coefficients. Then, the weight sets ω_1 and ω_2 can be linearized as:

$$\omega = a_1\omega_1^T + a_2\omega_2^T \tag{15}$$

② According to game theory, the linear combination coefficients α_1 and α_2 in Formula (11) are optimized and expressed as:

$$\min\left\|a_k\omega_k^T - \omega_k\right\|^2 \quad (k = 1,2) \tag{16}$$

③ According to the differential properties of the matrix, the linear differential equation group for optimizing the first derivative condition of formula (15) is:

$$\begin{bmatrix} \omega_1\omega_1^T & \omega_1\omega_2^T \\ \omega_2\omega_1^T & \omega_2\omega_2^T \end{bmatrix}\begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} \omega_1\omega_1^T \\ \omega_2\omega_2^T \end{bmatrix} \tag{17}$$

④ The optimal combination coefficients α_1 and α_2 are obtained via Formula (16). The normalization process is obtained as $a_1^* = a_1 / (a_1 + a_2)$ and $a_2^* = a_2 / (a_1 + a_2)$. Then, based on the game theory, the comprehensive weight ω can be obtained as:

$$\omega = a_1^*\omega_1^T + a_2^*\omega_2^T \tag{18}$$

The Construction of the Model Frame

To determine the quality level of water in the Tao Jiang River, their calculative flowchart is plotted in Fig. 2, and their specific procedure is listed as follows:

(1) The sample matrix of the origin matrix is constructed

(2) The decisive matrix of original data is established based on Equations (2) and (3).

(3) The optimal combination weight of different samples is obtained according to Equations (9)-(17)

(4) The target centers of different samples can be solved based on the decisive matrix and Equation (4)

(5) The positive and negative target center distances of different samples can be determined according to Equations (5) and (6).

(6) Determining different samples' synthetic target center distance based on Equation (7).

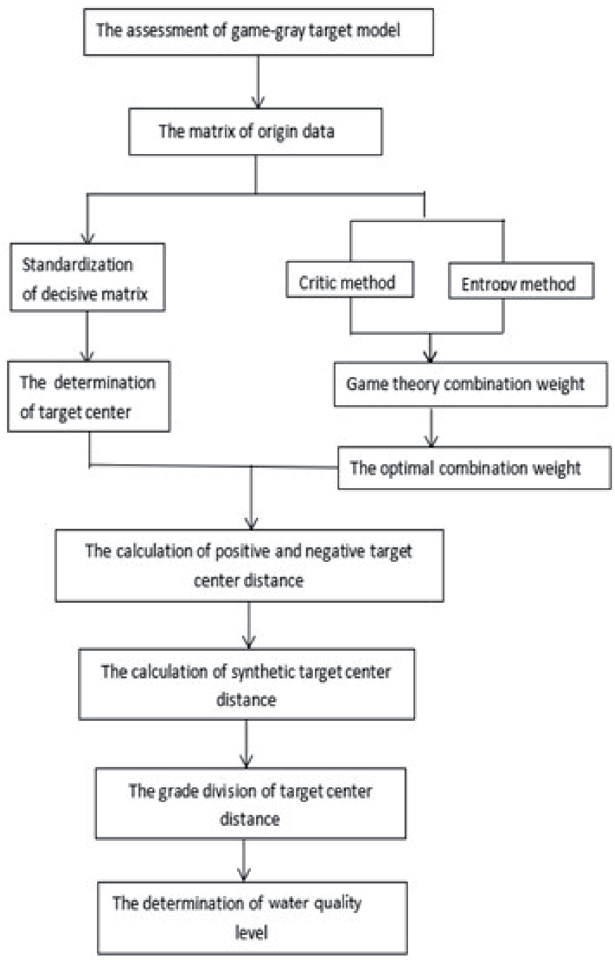

Fig. 2. Flowchart of the assessment framework.

(7) The partitioning of quality level is determined according to the target center distance range in combination with Equation (8).

(8) The final grade of water quality is determined.

Results and Discussion

The Assessment Procession

(1) The determination of the standard decisive matrix

Four indicators in Table 1 are all benefit types; based on Equations (1)-(3), and combined with Table 1, the decisive matrix is established, as shown in Table 2.

(2) The determination of the index weight

① Determining the weight coefficient ω_1 based on the entropy method

Based on Table 2, the corresponding weight coefficient can be obtained as:

$$\omega_1 = \begin{bmatrix} 0.1321 & 0.1446 & 0.1766 & 0.2945 & 0.1984 & 0.0537 \end{bmatrix}$$

② Calculating weight coefficient ω_2 based on the CRITIC method

According to Equations (9)-(11) and in conjunction with Table 1, the coefficients of correlation can be calculated as:

$$r = \begin{bmatrix} 1 & 0.9393 & 0.8979 & 0.7486 & 0.9417 & 0.9168 \\ 0.9393 & 1 & 0.962 & 0.8022 & 0.9627 & 0.9274 \\ 0.8979 & 0.962 & 1 & 0.8817 & 0.9662 & 0.934 \\ 0.7486 & 0.8022 & 0.8817 & 1 & 0.8172 & 0.8467 \\ 0.9417 & 0.9627 & 0.9662 & 0.8172 & 1 & 0.919 \\ 0.9168 & 0.9274 & 0.934 & 0.8467 & 0.919 & 1 \end{bmatrix}$$

Based on Equation (12), the standard deviation of different columns is obtained as:

$$C = \begin{bmatrix} 0.1821 & 0.1505 & 0.1227 & 0.2592 & 0.1401 & 0.1387 \end{bmatrix}$$

Similarly, Equation (13) calculates the weights of each evaluation index as:

$$\omega_2 = \begin{bmatrix} 0.1833 & 0.1515 & 0.1235 & 0.2609 & 0.1411 & 0.1396 \end{bmatrix}$$

③ Determination of combination weights

Based on Equations (14)-(17), the combination weight ω is obtained as:

$$\omega = \begin{bmatrix} 0.1288 & 0.1442 & 0.1801 & 0.2966 & 0.2021 & 0.0482 \end{bmatrix}$$

The view of weights is plotted in Fig. 3.

(3) The determination of the target center distance

The positive and negative target centers are, respectively, $x^{0+} = (1 \quad 0.88 \quad 0.67 \quad 0.3 \quad 0.85 \quad 0.82)$ and $x^{0-} = [-0.7 \quad -1 \quad -1 \quad -1 \quad -1 \quad -1]$. According to Equation (4), the interval distance between the positive and negative target centers is $d_0 = 1.6637$. According to Equations (5) and (6), the positive target center distance d^+ and negative target center distance d^- of different samples can be calculated, and their results are shown in Table 3.

According to Equation (7), the magnitude of the synthetic target center distance is shown in Table 4.

(4) The determination of the quality grade for water quality

The comprehensive target distance of each sample is arranged according to the quality grade; the critical

Table 2. The decisive matrix.

Sample sequence	X_1	X_2	X_3	X_4	X_5	X_6
1	0.94	0.81	0.51	0.29	0.81	0.52
2	1.00	0.88	0.67	0.30	0.85	0.82
3	0.32	0.55	0.62	0.23	0.77	0.31
4	0.24	0.65	0.54	0.23	0.54	0.29
5	0.10	0.37	0.19	0.15	0.12	0.52
6	0.16	0.22	0.11	0.16	0.23	0.12
7	-0.28	-0.17	-0.08	0.08	0.00	-0.13
8	-0.16	0.08	0.04	0.04	-0.31	-0.13
9	-0.37	-0.69	-0.40	-0.02	-0.50	-0.08
10	-0.47	-0.72	-0.25	-0.01	-0.50	-0.41
11	-0.77	-1.00	-1.00	-0.43	-1.00	-1.00
12	-0.71	-0.98	-0.96	-1.00	-0.92	-0.87

value of the target distance for each quality grade was obtained and is shown in Table 5. According to Equation (8), the target distance figure for each quality grade can be constructed; it is plotted in Fig. 4.

(5) The quality grade prediction of asphalt pavement

The Tao Jiang River is selected as an example in this paper to test the rationality and accuracy of the gray target evaluation model; the monitoring data for 13-18 samples are shown in Table 6.

The different parameters are substituted into the game gray target evaluation model, and the corresponding comprehensive target distance d_i^* is obtained. According to the different d_i^* ranges shown in Fig. 3, the quality grade of water is evaluated, and the results are shown in Table 7. The results obtained from the suggested model are compared with the other two methods, as shown in Fig. 5.

The game-gray target model is applied to evaluate the level of water quality. The assessment results are depicted in Table 7. It can be found in Table 7 that the grade of water quality in the Tao Jiang River watershed from Nos. 13–17# samples differ; the quality level at the No. 13# sample is II; one at the No. 14# sample is III; the rest are respectively IV, V, and VI. It means that the quality of the No. 13# sample is good, the quality of No. 14# is common, and the rest are inferior, bad, and worse, so its qualified rate is 40%. The corresponding remedial measures should be performed for the Nos. 15-17# samples section; no measurement needs to be adopted for other samples.

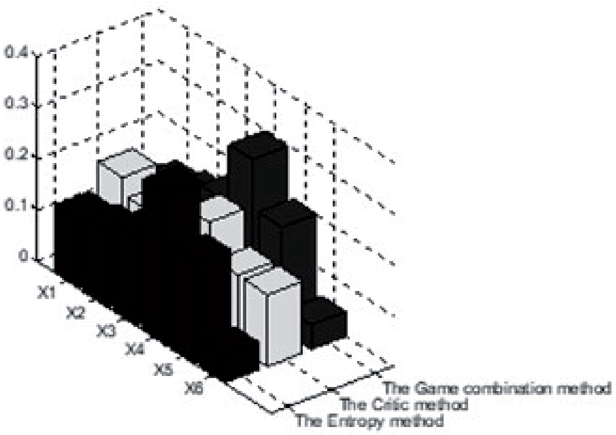

Fig. 3. The view of weight coefficients.

Table 3. The data of positive and negative target center distance.

Sample number	Positive target center distance d^+	Negative target center distance d^-
1	0.0346	0.9214
2	0	0.9556
3	0.2743	0.7067
4	0.2864	0.7239
5	0.3766	0.6067
6	0.392	0.571
7	0.6083	0.3609
8	0.5154	0.4649
9	0.7728	0.1856
10	0.8047	0.1513
11	0.9556	0
12	0.9357	0.0228

Table 4. The data on synthetic target center distance.

Sample	Synthetic target center distance d_i^*	Sample	Synthetic target center distance d_i^*
1	0.577	7	0.9039
2	0.5574	8	0.8467
3	0.7044	9	1.001
4	0.699	10	1.0196
5	0.7638	11	1.1063
6	0.7801	12	1.0948

Table 5. The critical value of synthetic target center distance.

Grade ranking	Critical value		
	g_f	c_f	h_f
B_1	0.577	0.5574	0.6407
B_2	0.7044	0.699	0.7341
B_3	0.7801	0.7638	0.8134
B_4	0.9039	0.8467	0.952
B_5	1.0196	1.001	1.0572
B_6	1.1063	1.0948	

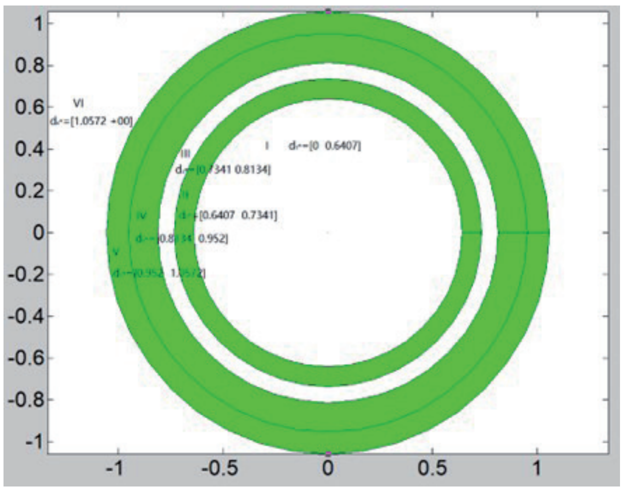

Fig. 4. The grade target distance for water quality.

Table 6. The monitoring data for the Tao River.

Sample sequence	X_1	X_2	X_3	X_4	X_5	X_6
13	10.67	0.76	0.28	0.08	0	0.36
14	6.92	2.86	0.59	0.46	0.09	0.19
15	5.5	4.64	3.39	0.57	0.16	0.99
16	3.73	9.36	4.05	1.33	0.27	1.16
17	2.49	13.86	8.96	1.78	0.36	1.1

Table 7. The synthetic target distance for predicting data.

Sample number	13	14	15	16	17
d_i^*	0.6613	0.7968	0.8661	0.9991	1.114
grade	II	III	IV	V	VI

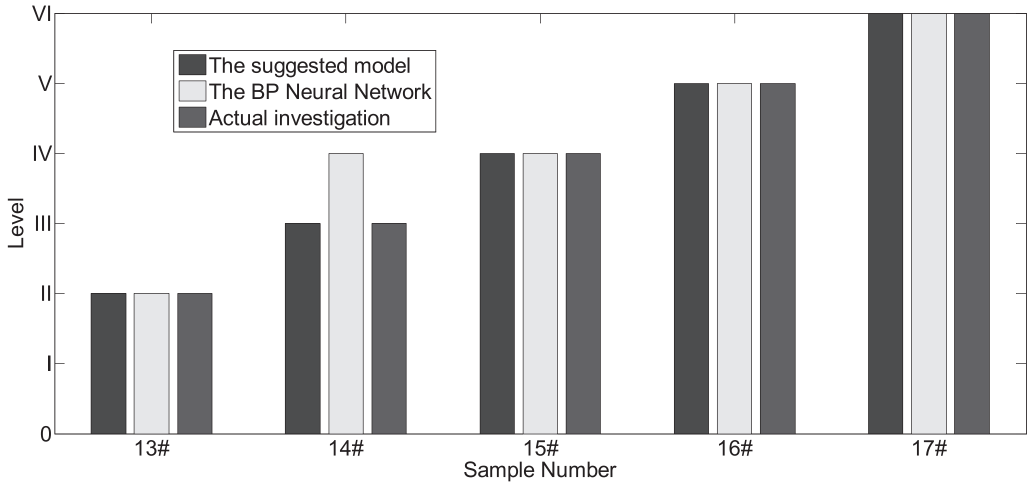

Fig. 5. The comparison of results from three methods.

The comparative results of the assessment model in Fig. 5 indicate that the proposed method is consistent with the investigations for five different samples. Its accuracy reaches 100%, which is more significant than the results from the BP Neural Network method (80%) [25]. Therefore, estimating water quality using the game-gray target model is feasible. The proposed approach provides additional details for assessing the level of water quality. For example, the quality level of No.7 is more likely to be level III than that of the No. 8 sample, as the synthetic target distance (0.9039) for level III is greater than the No.8 sample (0.8467). The results from the proposed model accurately demonstrate the water quality grade and further determine the risk grade rankings for different samples at the same level.

Discussion

The proposed method can accurately assess the water quality grade and has higher reliability and efficiency. However, its calculation process is complicated, and the randomness of evaluation indicators is not considered. So the proposed method can still be improved in the future. In future work, a three-dimensional gray target model will be applied, and it will be my future direction to assess the water quality level.

Conclusions

Considering the Dissolved oxygen (X_1), Permanganate index (X_2), BOD_5 (X_3), Ammonia nitrogen (X_4), Total phosphorus (X_5), and Fluoride (X_6), the game-gray target model is introduced in this manuscript to evaluate the level of water quality. Then, the level of water quality in five different samples is assessed according to the suggested model, and the final quality grade is determined.

Conclusions are obtained as follows:

(1) Because many factors affect water quality and a complex nonlinear relationship exists among different factors, a game combination weighting method

was applied to optimize the combination weights obtained from the Critic and entropy methods; the optimal weighting coefficients were obtained. In combination with the gray target decision theory, a game gray target evaluation model is established to realize the quantitative evaluation of water quality grade.

(2) The level of water quality in the Tao Jiang River watershed was evaluated using the game gray target evaluation model; the evaluation result is consistent with the actual investigation; its accuracy reaches 100%, more significant than the results from the BP Neural Network method (80%). This demonstrates that the suggested model is feasible. It has higher validity and accuracy in evaluating water quality and has better practical application prospects.

(3) The game gray target evaluation model can mine the small sample data to the greatest extent and can weaken the information shortage problem caused by the small sample to a certain extent. The final quality grade of water can be evaluated quantitatively.

(4) At present, the gray target decision-making theory is only applied preliminarily to the water quality prediction, and further research is needed, for example, on how to ensure the comprehensiveness and reliability of the water quality index and how to get a more accurate and reliable index weight, etc. Therefore, the suggested model has great potential for improvement in the future.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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References

1. GU X.B., WU S.T., JI X.J., ZHU Y.H. The Risk Assessment of Debris Flow Hazards in Banshanmen Gully Based on the Entropy Weight-Normal Cloud Method. Advances in Civil Engineering, **1**, 1, **2021**.
2. GU X.B., WU Q.H., ZHU Y.H. The experimental investigation on the propagation process of crack for brittle rock similar material. Geotechnical and Geological Engineering, **37**, 4714, **2019**.
3. GU X.B., WU Q.H. Seismic stability analysis of waterfront rock slopes using the modified pseudo-dynamic method. Geotechnical and Geological Engineering, **37**, 1741, **2019**.
4. SINGH V.P. Entropy theory and its application in environmental and water engineering. John Wiley & Sons, **662**, 332, **2020**.
5. TYAGI S., SHARMA B., SINGH P. Water quality assessment in terms of water quality index. American Journal of Water Resources, **1**, 34, **2013**.
6. LUMB A., SHARMA T.C., BIBEAULT J.F.A. Review of genesis and evolution of water quality index (WQI) and some future directions. Water Quality, Exposure and Health, **3**, 11, **2011**.
7. LIU E.K., MEI X.R., YAN C.R. Effects of water stress on photosynthetic characteristics, dry matter translocation and WUE in two winter wheat genotypes. Agriculture Water Management, **167**, 75, **2016**.
8. YAN C.A., ZHANG W., ZHANG Z. Assessment of water quality and identification of polluted risky regions based on field observations & GIS in the Honghe river watershed, China. PloS one, **10**, 119, **2015**.
9. NING L., LIYUAN Y., JIRUI D. Heavy metal pollution in surface water of Linglong gold mining area, China. Procedia Environmental Sciences, **10**, 914, **2011**.
10. IKPE N.C., KENECHUKWU E.C., IKECHUKWU E.C. Use of integrated pollution indices in assessing heavy metals pollution in soils of three auto mechanic villages in Abuja. African Journal of Environmental Science and Technology, **12**, 370, **2018**.
11. ZHU Y.H., LIU X.R., ZHOU J.P. Rock burst prediction analysis based on SVR algorithm. Journal of China Coal Society, **33**, 277, **2008**.
12. ZHOU X.P., GU X.B., QIAN Q.H. Seismic bearing capacity of shallow foundations resting on rock masses subjected to seismic loads. KSCE Journal of Civil Engineering, **20**, 216, **2016**.
13. ZHOU X.P., PAN X.K., CHENG H. The nonlinear creep behaviors of sandstone under the different confining pressures based on NMR Technology. Rock Mechanics and Rock Engineering, **54**, 4889, **2021**.
14. XU H., LI S.C., LI L.P. Risk assessment of water or mud inrush of karst tunnels based on analytic hierarchy process. Rock and Soil Mechanisms, **32**, 1757, **2011**.
15. ZHANG Y.G., XIE Y.L., ZHANG Y., QIU J.B., WU S.X. The adoption of deep neural network (DNN) to the prediction of soil liquefaction based on shear wave velocity. Bulletin of Engineering Geology and the Environment, **32**, 457, **2021**.
16. GU X.B., SHAO J.L., WU S.T., WU Q.H., BAI H. The Risk Assessment of Debris Flow Hazards in Zhouqu Based on the Projection Pursuit Classification Model. Geotechnical and Geological Engineering, **8**, 4, **2021**.
17. GU X.B., MA Y., WU Q.H. The risk assessment of landslide hazards in Shiwangmiao based on intuitionistic fuzzy sets-Topsis model. Natural Hazards, **32**, 45, **2021**.

18. GUO G.L, GUO R., GU J.F. Application of gray theory in hazard assessment of debris flow disaster in zhouqu nanyu gully. Arid Land Geography, **86**, 33, **2019**.
19. HOLLINGSWORTH R., KOVACS G.S. Soil Slumps and Debris Flows: Prediction and Protection. Bulletin of the Association of Engineering Geologists, **38**, 17, **198**1.
20. ZHAO Y., BI J., WANG C.L, LIU P.F. Effect of Unloading Rate on the Mechanical Behavior and Fracture Characteristics of Sandstones Under Complex Triaxial Stress Conditions. Rock Mechanics And Rock Engineering, **54** ,4851, **2021**.
21. WU X., WANG H., HE Y. Excavation stability evaluation of Karst tunnel based on fuzzy matter-element. China Safety Science Journal, **28**, 99, **2018**.
22. GU X.B., WANG L., WU Q.H. The Risk Assessment of Debris Flow in the Duba River Watershed Using Intuitionistic Fuzzy Sets: TOPSIS Model. Mathematical Problems in Engineering, **20**, 3251, **2022**.
23. CHEN J.W., ZHOU X.P. The enhanced extended finite element method for the propagation of complex branched cracks. Engineering Analysis with Boundary Elements, **104**, 46, **2019**.
24. CHEN J., ZHOU X. Advanced absorbing boundaries for elastodynamic finite element analysis: The added degree of freedom method. Computer Methods In Applied Mechanics And Engineering, **420**, 116, **2024**.
25. XU Y.Y. Water Quality Evaluation and Cause Analysis of Tao jiang River Based on BP Neural Network and WPI. Jiangxi University of Science and Technology, **32**, 115, **2021.**