











The expected payoff function for an academic institution choosing the “no synergy” strategy is:

$$U_{z2} = xy(L_2 + \rho_1 R_1 - \eta F - P) + x(1-y)(L_2 - \eta F) + (1-x)y(L_2 + \rho_1 R_1 - P) + (1-x)(1-y)L_2 \quad (8)$$

The average expected return function for academic and research organizations is:

$$\bar{U}(z) = zU_{z1} + (1-z)U_{z2} \quad (9)$$

### Stability Analysis of the Replicated Dynamic Equations for the Subjects of the Three-Party Game

A system of replicated dynamic equations is constructed from Eqs. (1)-(9) to obtain Eqs. (10)-(12).

$$\begin{cases} F(x) = \frac{dx}{dt} = x(U_{x1} - \bar{U}_x) = x(1-x)(U_{x1} - U_{x2}) \\ = x(1-x)[W + 2\eta F - (y+z)(T + S_1 + \eta F) + yz(T + 2S_1 - S)] \end{cases} \quad (10)$$

$$\begin{cases} F(y) = \frac{dy}{dt} = y(U_{y1} - \bar{U}_y) = y(1-y)(U_{y1} - U_{y2}) = \\ y(1-y)\{P + (1-z)(R_1 - C_1) - z[\gamma\alpha(C_1 + C_2) + \mu\beta(R_1 + R_2) \\ - \rho_2 R_2] + x(S_1 + \eta F) + xz(\varphi S - S_1)\} \end{cases} \quad (11)$$

$$\begin{cases} F(z) = \frac{dz}{dt} = z(U_{z1} - \bar{U}_z) = z(1-z)(U_{z1} - U_{z2}) \\ = z(1-z)\{y[\mu(1-\beta)(R_1 + R_2) - \gamma(1-\alpha)(C_1 + C_2) - \rho_1 R_1] + P + (1-y) \\ (R_2 - C_2) + x(S_1 + \eta F) + xy[(1-\varphi)S - S_1]\} \end{cases} \quad (12)$$

### Stability Analysis of Government Gaming Strategies

A first-order partial derivative concerning the variable  $x$  in (10) yields:

$$F'(x) = (1-2x)[W + 2\eta F - (y+z)(S_1 + T + \eta F) + yz(2S_1 + T + S)] \quad (13)$$

According to equation (13), let  $y^* = \frac{z(T + S_1 + \eta F) - W - 2\eta F}{z(T + 2S_1 - S) - T - S_1 - \eta F}$ , it can be seen that

(1) when  $y = y^*$ ,  $F(x) \equiv 0$ , so any  $x \in [0, 1]$  is a stable point; (2) When  $y \neq y^*$ , two equilibrium points  $x = 0$  and  $x = 1$  are obtained from  $F(x) \equiv 0$  as the two steady state points of the government game strategy in the system.

**Lemma 1:** When  $y < y^*$ , the government evolutionary stabilization strategy is  $x = 1$ ; When  $y > y^*$ , the government evolutionary stabilization strategy is  $x = 0$ .

**Proof:**

According to the stability theorem of differential equations, if the government’s evolutionary stabilization strategy is to “participate” in the green innovation ecosystem, then it needs to satisfy the following criteria, then it needs to satisfy:  $F(x) = 0$ , and  $F'(x) < 0$  such that:

$$G(y) = W + 2\eta F - (y+z)(S_1 + T + \eta F) + yz(2S_1 + T + S) \quad (14)$$

$$\text{then } \frac{dG(y)}{dy} = -\eta F - zS - (1-2z)S_1 - (1-z)T < 0 \quad (15)$$

Thus,  $G(y)$  is a decreasing function with respect to  $y$ . When  $y < y^*$ ,  $G(y) > 0$ ,  $F'(x)|_{x=0} > 0$ ,  $F'(x)|_{x=1} < 0$ ,  $x = 1$  is an evolutionary stabilization point. When  $y > y^*$ ,  $G(y) < 0$ ,  $F'(x)|_{x=1} > 0$ ,  $F'(x)|_{x=0} < 0$ ,  $x = 0$  is an evolutionary stabilization point. Based on the above analysis, a phase diagram of the government’s strategy choices can be obtained, as shown in Fig. 1. The space in Fig. 1 is divided into two parts by the surface  $y = y^*$ , whose volumes are denoted as  $V_{x1}$  and  $V_{x2}$ , representing the probability that the government chooses the “participate” strategy and the likelihood that it determines the “don’t participate” strategy, respectively.

**Corollary 1:** When the initial state of the government’s decision is located in space  $V_{x1}$ ,  $x = 1$  is a stable equilibrium point in space  $V_{x1}$ , i.e., the government’s game strategy gradually evolves in the direction of “participating” in the green innovation ecosystem. When the initial state of the government’s decision is located in space  $V_{x2}$ ,  $x = 0$  is a stable equilibrium point in space  $V_{x2}$ , i.e., the government’s game strategy gradually evolves in the direction of “non-participation” in the green innovation ecosystem.

**Corollary 2: Parametric analysis.** As can be seen from Fig. 1, when  $W$ ,  $\eta F$  becomes larger (or  $S_1$ ,  $T$ ,  $S$  becomes smaller), and other parameters remain unchanged,  $y^*$  becomes larger, the cross-section part moves in the positive direction of the  $y$ -axis, the space  $V_{x1}$  becomes more extensive, the area  $V_{x2}$  becomes smaller, and the probability of the government’s strategic choice tending to be “participation” becomes more considerable.

**End of proof.**



equilibrium points  $z = 0$  and  $z = 1$  are obtained from  $F(z) \equiv 0$  as the two steady state points of the core firm's game strategy in the system.

**Lemma 3:** When  $x < x^{**}$ , the evolutionary stabilization strategy for academic and research institutions is  $z = 0$ ; When  $x > x^{**}$ , the evolutionary stabilization strategy for academic and research institutions is  $z = 1$ .

**Proof:**

According to the stability theorem of differential equations, if "synergy" is an evolutionarily stable state for the strategy choice of academic and research institutions, then it needs to be satisfied:  $F(z) \equiv 0$ , and  $F'(z) < 0$  such that:

$$G(x) = P + (1-y)(R_2 - C_2) + y[\mu(1-\beta)(R_1 + R_2) - \gamma(1-\alpha)(C_1 + C_2) - \rho_1 R_1] + x(S_1 + \eta F) + xy[(1-\varphi)S - S_1] \quad (20)$$

then

$$\frac{dG(x)}{dx} = S_1 + \eta F + y[(1-y)S - S_1] > 0 \quad (21)$$

Therefore,  $G(x)$  is an increasing function about  $x$ . When  $x < x^{**}$ ,  $G(x) < 0$ ,  $F'(z)|_{z=1} > 0$ ,  $F'(z)|_{z=0} < 0$ ,  $z = 0$  are evolutionarily stable points. When  $x > x^{**}$ ,  $G(x) > 0$ ,  $F'(z)|_{z=0} > 0$ ,  $F'(z)|_{z=1} < 0$ ,  $z = 1$  are evolutionarily stable points.

Corollary: Similarly, the analysis of the phase diagram of the evolution of the decision-making behavior of academic and research institutions shows that when  $C_1, C_2, \rho_1 R_1$  becomes larger (or  $S_1, R_1, R_2, S, \eta F, P$  becomes smaller) and the other parameters remain constant,  $x^{**}$  becomes more prominent, the probability of "no synergy" in the strategic choices of academic and research institutions has become higher.

**End of proof.**

### Stability Analysis of Game System Combinatorial Strategies

To further explore the strategic evolutionary equilibrium point of core enterprises, academic and research institutions, and the government in the green innovation ecosystem. According to Friedman's [19] evolutionary game analysis method, the local stability analysis of the Jacobi matrix of this system obtains the evolutionary stability strategy of the three-dimensional dynamical system, and the partial derivatives of  $x, y$ , and  $z$  for the replicated dynamical system (10)-(12) of the three parties can be obtained as follows for the three-party subject's Jacobi matrix J:

$$J = \begin{pmatrix} \frac{dF(x)}{dx} & \frac{dF(x)}{dy} & \frac{dF(x)}{dz} \\ \frac{dF(y)}{dx} & \frac{dF(y)}{dy} & \frac{dF(y)}{dz} \\ \frac{dF(z)}{dx} & \frac{dF(z)}{dy} & \frac{dF(z)}{dz} \end{pmatrix} \quad (22)$$

$$= \begin{pmatrix} (1-2x) \begin{bmatrix} W - (y+z)(S_1 + T + \eta F) \\ +yz(2S_1 + T + S) + 2\eta F \end{bmatrix} \\ y(1-y) [\eta F + z\varphi S + (1-z)S_1] \\ z(1-z) [\eta F + (1-y)S_1 + y(1-\varphi)S] \\ x(1-x) [(2z-1)S_1 + (z-1)T - \eta F - zS] \\ (1-2y) \begin{bmatrix} P + (1-z)(R_1 - C_1) - \rho_2 R_2 \\ -z[\gamma\alpha(C_1 + C_2) + \mu\beta(R_1 + R_2)] \\ +x(S_1 + \eta F) + xz(\varphi S - S_1) \end{bmatrix} \\ z(1-z) \begin{bmatrix} \mu(1-\beta)(R_1 + R_2) + x(S - \varphi S - S_1) \\ +C_2 - \rho_1 R_1 - \gamma(1-\alpha)(C_1 + C_2) - R_2 \end{bmatrix} \\ x(1-x) [(y-1)T + (2y-1)S_1 - \eta F - yS] \\ y(1-y) \begin{bmatrix} C_1 - \gamma\alpha(C_1 + C_2) + \mu\beta(R_1 + R_2) \\ +x(\varphi S - S_1) - \rho_2 R_2 - R_1 \end{bmatrix} \\ (1-2z) \begin{bmatrix} (1-y)R_2 + y\mu(1-\beta)(R_1 + R_2) \\ +x(1-y)S_1 + xy(1-\varphi)S + P + x\eta F \\ -(1-y)C_2 - y\gamma(1-\alpha)(C_1 + C_2) - y\rho_1 R_1 \end{bmatrix} \end{pmatrix}$$

According to et al. Ritzberger [20] and Selten [21], in multiple swarm evolutionary games, a strict Nash equilibrium is a stable solution of the evolutionary game, and that strict Nash equilibrium is pure strategy. In Eq. (16), the nine local equilibrium points  $E_1(0, 0, 0)$ ,  $E_2(0, 0, 1)$ ,  $E_3(0, 1, 0)$ ,  $E_4(0, 1, 1)$ ,  $E_5(1, 0, 0)$ ,  $E_6(1, 0, 1)$ ,  $E_7(1, 1, 0)$ ,  $E_8(1, 1, 1)$ ,  $E_9(x^*, y^*, z^*)$  of the replicated dynamical system (16) can be obtained by letting  $F(x) = 0, F(y) = 0, F(z) = 0$ . The system's stability is further analyzed by substituting each of the above equilibrium points into the Jacobi matrix to obtain the eigenvalues of the corresponding Jacobi matrix. Firstly, taking the equilibrium point  $E(1, 1, 1)$  as an example, the Jacobi matrix of this point is obtained as:



$$J_8 = \begin{pmatrix} -(W-T-S) & 0 & 0 \\ 0 & -\begin{bmatrix} \mu\beta(R_1+R_2)+\varphi S+\eta F \\ +P-\gamma\alpha(C_1+C_2)-\rho_2 R_2 \end{bmatrix} & 0 \\ 0 & 0 & -\begin{bmatrix} \mu(1-\beta)(R_1+R_2)+(1-\varphi)S+P \\ +\eta F-\gamma(1-\alpha)(C_1+C_2)-\rho_1 R_1 \end{bmatrix} \end{pmatrix} \quad (23)$$

It follows that the eigenvalues of the Jacobi matrix at this point are  $\lambda_1 = -(W - T - S)$ ;  $\lambda_2 = -[\mu\beta(R_1 + R_2) + \varphi S + \eta F + P - \gamma\alpha(C_1 + C_2) - \rho_2 R_2]$ ;  $\lambda_3 = -[\mu(1 - \beta)(R_1 + R_2) + (1 - \varphi)S + P + \eta F - \gamma(1 - \alpha)(C_1 + C_2) - \rho_1 R_1]$ .

According to the determination method proposed by Friedman, the stable point of the replicated equilibrium equation is the stabilizing strategy (ESS) if the eigenvalues are all negative, and vice versa for the unstable end. The positivity or negativity of the determinant  $DetJ$  and trace  $TrJ$  of the matrix  $J$  can also determine the stability of the equilibrium point of a system of differential equations. The stabilization point of the replicated equilibrium equation is the stabilization strategy (ESS) when the matrix  $J$  has  $DetJ > 0$  and  $TrJ < 0$  [22]. Then, the stabilization point of each party's innovation strategy will be  $E(1, 1, 1)$ . At this time, the influence factors of collaborative innovation strategies between the subjects should meet the following situation:

$$\begin{cases} W > T + S \\ \mu\beta(R_1 + R_2) + \varphi S + \eta F + P > \gamma\alpha(C_1 + C_2) + \rho_2 R_2 \\ \mu(1 - \beta)(R_1 + R_2) + (1 - \varphi)S + P + \eta F > \gamma(1 - \alpha)(C_1 + C_2) + \rho_1 R_1 \end{cases} \quad (24)$$

As shown in Equation (24), at this point, the benefit of the government participation strategy is greater than the sum of the regulatory cost of its government participation strategy and the incentive subsidy given by the government for collaborative R&D and innovation; the sum of government subsidies received by core firms, synergy gains, government penalties for core firms' betrayal of the contract, and the amount of liquidated damages for core firms' betrayal of the contract is greater than the sum of the synergy costs of the core firms and the green technology spillovers that the core firms receive from exiting the green innovation ecosystem; the sum of the government subsidies received by the academic and research institutions, the synergy benefits, the government's penalty for the betrayal of the contract by the educational and research institutions, and the liquidated damages for the betrayal of the contract by the academic and research institutions is greater than the sum of the synergy costs of the core firms and the green technology spillovers that the core firms receive by exiting the green innovation ecosystem.

To facilitate the study of whether the other eight equilibrium points satisfy the evolutionary steady state, and for the sake of non-generality, the correlation coefficients  $W - T - S > 0$ ,

$$\begin{aligned} & \mu\beta(R_1 + R_2) + \varphi S + \eta F + P - \gamma\alpha(C_1 + C_2) - \rho_2 R_2 > 0 \\ & \mu(1 - \beta)(R_1 + R_2) + (1 - \varphi)S + P + \eta F - \gamma(1 - \alpha)(C_1 + C_2) - \rho_1 R_1 > 0 \end{aligned},$$

are assumed, and the sign of the eigenvalues corresponding to the other eight equilibrium points is obtained according to the analytical method described above, as shown in Table 3.

The above analysis shows that  $E_2E_4E_7$  may be the gradual stability point of the system. Government subsidies and penalties for green innovation for core enterprises and academic and research institutions, as

Table 3. Local stability analysis of each equilibrium point.

Equilibrium	Eigenvalue			Stability
	$\lambda_1$	$\lambda_2$	$\lambda_3$	
$E_1(0, 0, 0)$	$> 0$	$> 0$	$> 0$	Saddle point
$E_2(0, 0, 1)$	—	—	$< 0$	When $\frac{T+S}{\gamma\alpha(C_1+C_2)+\rho_2R_2} > \frac{W+\eta F}{\mu\beta(R_1+R_2)+P}$ , the point is stable.
$E_3(0, 1, 0)$	—	$< 0$	—	Saddle point
$E_4(0, 1, 1)$	—	$< 0$	$< 0$	When $T + S > W$ , the point is stable.
$E_5(1, 0, 0)$	$< 0$	$> 0$	$> 0$	Destabilization point
$E_6(1, 0, 1)$	$< 0$	—	$< 0$	Saddle point
$E_7(1, 1, 0)$	$< 0$	$< 0$	—	When $\frac{\gamma(1-\alpha)(C_1+C_2)+\rho_1R_1}{\mu(1-\beta)(R_1+R_2)+P+(1-\varphi)S+\eta F} > 1$ , the point is stable.
$E_8(1, 1, 1)$	$< 0$	$< 0$	$< 0$	ESS
$E(x^*, y^*, z^*)$	$DetJ > 0 \cap TrJ = 0$			Saddle point







