

Original Research

Fuzzy Attribute Interval Modeling for Risk Evaluation of Water Inrush in Deep and Long Tunnels and Engineering Applications

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Abstract

Water inrush is one of the most frequent and harmful geological hazards during tunnel construction, especially in deep-buried and long tunnels. Given the complexity and uncertainty of the geological conditions along the deep-buried and long tunnels, a small-scale mathematical interval is used to quantify the evaluation indices rather than a certain value, and an attribute interval assessment method for tunnel water inrush is proposed. Firstly, considering the hazard-pregnant and hazard-causing factors of water inrush occurrence, the formation lithology, unfavorable geology, groundwater level, topography and geomorphology, attitude of rock formation, contact zone of dissolvable and insoluble rocks, and layer and interlayer fissures are selected as the evaluation indices. Then, the single-index attribute measure functions are constructed to calculate the upper and lower limits of each evaluation index belonging to the four risk levels. The fusion function of multi-index attribute measure intervals is established, and the most probable risk level is identified. Meanwhile, a new comprehensive weighting method for risk assessment of tunnel water inrush is presented by combining the frequency statistics method and triangular fuzzy number theory-analytic hierarchy process (TFN-AHP). Finally, the proposed method is applied to the Yunwushan Tunnel. The evaluation results agree well with the actual situation, which verifies the practicality and feasibility of this method and provides a basis for the risk control of geological hazards in tunnel engineering.

Keywords: deep-buried and long tunnel, water inrush, risk assessment, attribute mathematical theory, engineering application

Introduction

At present, tunnel engineering is gradually developing towards the trend of large buried depths and long lines. In the process of tunnel construction, problems such as high ground stress, high water pressure, and complex geological conditions have become increasingly prominent, resulting in increased construction difficulty and frequent geological hazards, especially water and mud inrush [1, 2]. According to statistics, nearly 80% of the tunnels have experienced water inrush [1, 3]. Due to the limitation of the geological survey technology, it is difficult to find water-rich faults, water-filled caves, and other unfavorable karst structures along the tunnel, which further increases the risk of water inrush occurrence.

To effectively control the occurrence of water and mud inrush, scholars at home and abroad have carried out a lot of research on the risk assessment of water inrush in tunnels [4, 5]. Zhou et al. [6] and Wang et al. [7] established an attribute recognition model of water inrush risk assessment based on the attribute mathematical theory. Considering the complexity and uncertainty of geological conditions, Li et al. [8] and Wang et al. [9] proposed an attribute interval recognition model to systematically evaluate the risk of water inrush in karst tunnels. Wang et al. [10] presented the risk evaluation model of water inrush with set pair analysis. However, the connection degree of the evaluation indices of set pair analysis is linear, so Jiang et al. [11] proposed a cloud model set pair analysis method to optimize the connection degree. Yuan et al. [12] and Zhou et al. [13] established an improved grey evaluation method based on grey system theory. Wang et al. [14] developed an interval risk assessment method based on the analytic hierarchy process and fuzzy mathematical theory and proposed a three-stage risk assessment model of water inflow and inrush in karst tunnels. Wang et al. [15] improved the state variable weight theory to determine the index weight, and established risk assessment of tunnel water inrush based on the ideal point method and matter element theory. Wang et al. [16] put forward a novel model based on the ideal point for risk assessment of water inrush. Aiming at the fuzziness of geological conditions and ideal points, Wang et al. [17] constructed a new ideal point interval recognition model for risk assessment of water inrush. Zhang et al. [18, 19] presented an assessment model based on extension theory to evaluate the risk of water inrush. Li et al. [20] developed a software system for risk assessment of water inrush considering risk factors and fuzzy mathematics. Li et al. [21] developed a simple and practical software package based on the proposed Attribute Interval Evaluation Theory (AIET), which can automatically complete the risk assessment process. The third is multi-stage dynamic risk assessment. Li et al. [22] established a two-stage risk evaluation method for water inrush based on FAHP to carry out the preliminary evaluation at the prospecting

and design stage and dynamic evaluation at the construction stage. Xu et al. [23] proposed a three-stage risk assessment and management method, including the preliminary, secondary and dynamic assessment of water inrush risk. Wang et al. [24] presented a dynamic risk assessment methodology based on the attribute interval assessment model, which consists of pre-assessment before excavation and post-assessment after excavation and before primary support. The above research results have played a positive role in preventing and controlling the occurrence of tunnel water inrush, but there are still the following limitations: (1) A certain value is used to quantify the evaluation index, which leads to the loss of effective information. (2) The weight distribution of the evaluation indices is based on the subjective weighting method, which is easily affected by the personal preference and knowledge level of the experts.

Based on the research results of the references [25-28], a comprehensive weighting method is established to determine the weights of the evaluation indices and an improved fuzzy attribute interval assessment model is proposed, which is used to evaluate the water inrush risk of the Yunwushan Tunnel of Chongqing suburban railway from Bishan to Tongliang. The evaluation results are compared with the actual situation and other assessment methods to verify the practicability and feasibility of the proposed method.

Material and Methods

Attribute Fuzzy Interval Assessment Model

Let an evaluation object has n evaluation indices I_i ($i = 1, 2, \dots, n$), and each index I_i has K risk assessment levels C_k ($k = 1, 2, \dots, K$). Considering the complexity of geological conditions and the uncertainty of the values of the influencing factors, the value of the evaluation index I_i is quantified by a small range interval $[t_{ix}, t_{iy}]$.

Single-Index Attribute Measure Analysis

The single-index attribute measure μ_{xik} is used to represent the attribute value of the measured interval values $[t_{ix}, t_{iy}]$ of the index I_i belonging to the risk level C_k . According to the classification criteria presented

Table 1. Risk grading criteria of single evaluation index.

Evaluation index I_i	Risk level			
	C_1	C_2	...	C_K
I_1	$a_{11} \sim b_{11}$	$a_{12} \sim b_{12}$...	$a_{1K} \sim b_{1K}$
I_2	$a_{21} \sim b_{21}$	$a_{22} \sim b_{22}$...	$a_{2K} \sim b_{2K}$
...
I_n	$a_{n1} \sim b_{n1}$	$a_{n2} \sim b_{n2}$...	$a_{nK} \sim b_{nK}$

in Table 1, the attribute matrix A and B are defined by Eq. (1) ~ Eq. (2).

$$A = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1K} \\ a_{21} & a_{22} & \dots & a_{2K} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \dots & a_{nK} \end{bmatrix} \quad (1)$$

$$B = \begin{bmatrix} b_{11} & b_{12} & \dots & b_{1K} \\ b_{21} & b_{22} & \dots & b_{2K} \\ \vdots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \dots & b_{nK} \end{bmatrix} \quad (2)$$

In the Equations, when the greater the value of the evaluation index, the higher the risk level, it satisfies $a_{ij} < b_{ij}$, satisfy $a_{i1} < a_{i2} < \dots < a_{iK}$, $b_{i1} < b_{i2} < \dots < b_{iK}$. When the smaller the value of the evaluation index, the higher the risk level, it satisfies $a_{ij} > b_{ij}$, $a_{i1} > a_{i2} > \dots > a_{iK}$, $b_{i1} < b_{i2} < \dots < b_{iK}$.

When $a_{jk} < b_{jk}$, $a_{i1} < a_{i2} < \dots < a_{iK}$ and $b_{i1} < b_{i2} < \dots < b_{iK}$, the calculation formulas of the single-index attribute measure for matrix A are as follows:

$$\mu_{i1}(t_i) = \begin{cases} 1 & t_i \leq a_{i1} \\ 0 & t_i > a_{i1} \end{cases} \quad (3)$$

$$\mu_{ik}(t_i) = \begin{cases} 0 & t_i < a_{ik} \\ \frac{a_{ik+1} - t_i}{a_{ik+1} - a_{ik}} & a_{ik} \leq t_i \leq a_{ik+1} \\ 0 & t_i > a_{ik+1} \end{cases} \quad (4)$$

$$\mu_{ik+1}(t_i) = \begin{cases} 0 & t_i \leq a_{ik} \\ \frac{t_i - a_{ik}}{a_{ik+1} - a_{ik}} & a_{ik} < t_i \leq a_{ik+1} \\ \frac{a_{ik+2} - t_i}{a_{ik+2} - a_{ik+1}} & a_{ik+1} < t_i < a_{ik+2} \\ 0 & t_i \geq a_{ik+2} \end{cases} \quad (5)$$

$$\mu_{iK}(t_i) = \begin{cases} 0 & t_i \leq a_{iK-1} \\ \frac{t_i - a_{iK-1}}{a_{iK} - a_{iK-1}} & a_{iK-1} < t_i < a_{iK} \\ 1 & t_i \geq a_{iK} \end{cases} \quad (6)$$

Similarly, the single-index attribute measure function of matrix B can be obtained. The measured lower limit value t_{ix} and upper limit value t_{iy} of the index I_i are substituted into Eq.(4) ~ Eq.(6), then 4 matrices

of single-index attribute measure can be obtained, and their specific form can be referred to the reference [9, 29].

Synthetic Attribute Measure Analysis

Since each evaluation index has four vectors of single-index attribute measure $\underline{\mu}_{ixk}$, $\bar{\mu}_{ixk}$, $\underline{\mu}_{iyk}$, and $\bar{\mu}_{iyk}$, the synthetic attribute measure has 4^n combinations. To simplify the calculation of the synthetic attribute measures, the average value μ_{ik} of four single-index attribute measures is calculated.

$$\mu_{ik} = \frac{\underline{\mu}_{ixk} + \bar{\mu}_{ixk} + \underline{\mu}_{iyk} + \bar{\mu}_{iyk}}{4} \quad (7)$$

Then, the single-index attribute measure of each evaluation index is weighted and summed to determine the synthetic attribute measure μ_k .

$$\mu_k = \sum_{i=1}^n w_i \cdot \mu_{ik} \quad (8)$$

Where w_i is the weight of the evaluation indicator I_i , $0 \leq w_i \leq 1$, and $\sum_{i=1}^n w_i = 1$.

Attribute Recognition Analysis

Based on the obtained synthetic attribute measure, the confidence criterion is used to identify the most probable risk level of the evaluation object. The confidence level λ is between 0.6 and 0.7. Assuming that the risk assessment comment set is ordered, the risk level of the object to be evaluated is determined as follows:

When $C_1 < C_2 < \dots < C_K$,

$$k_0 = \max \left\{ k : \sum_{l=k}^K u_{xl} \geq \lambda, 1 \leq k \leq K \right\} \quad (9)$$

When $C_1 > C_2 > \dots > C_K$,

$$k_0 = \min \left\{ k : \sum_{l=1}^k u_{xl} \geq \lambda, 1 \leq k \leq K \right\} \quad (10)$$

No matter what kind of ordered form, as long as Eq. (9) or Eq. (10) is satisfied, the risk level of the object to be evaluated is C_{k_0} .

Combination Weighting Method

The weights of the evaluation indices have a direct impact on the accuracy and rationality of the assessment results and are crucial to the risk level of the object to be evaluated. To give full play to the subjective initiative

of experts and take into account the actual situation of the project site, a combination weighting method based on the frequency statistics method and triangular fuzzy number-analytic hierarchy process (TFN-AHP) is used to determine the objective weight and subjective weight of the evaluation index respectively. Finally, the comprehensive weight is calculated as follows:

$$w_i = c_1 w_i^s + c_2 w_i^o \tag{11}$$

Where w_i^s is the subjective weight of the evaluation index I_i , determined by the TFN-AHP method; w_i^o is the objective weight of the evaluation index I_i , determined by the frequency statistics method. c_1 and c_2 are the distribution coefficients of the objective weight and subjective weight respectively, which satisfy $c_1 + c_2 = 1$, and the specific values are determined by the experts according to the field situation.

Subjective Weighting Method Based on TFN-AHP

The AHP mainly relies on the subjective judgment of experts, but is limited by the experts' knowledge level, resulting in ambiguity and hesitation in the comparative analysis of the relative importance of evaluation indices. Therefore, the TNF is introduced to improve the AHP, and the triangular fuzzy number $R_{ij} = (r_{ij}^l, r_{ij}^m, r_{ij}^u)$ is used to quantify the relative importance between the evaluation indices, in which r^l, r^m, r^u represent the lower limit, the most probable value, and the upper limit of the relative importance between the evaluation indices respectively. The specific triangular fuzzy scaling method is shown in Table 2.

An n -order judgment matrix can be formed from R_{ij} ($i, j = 1, \dots, n$), namely $R = (R_{ij})_{n \times n}$. To test the consistency of the judgment matrix R , the triangular fuzzy opinions should be aggregated first.

$$m_{ij} = \frac{r_{ij}^L + 2r_{ij}^M + r_{ij}^U}{4} \quad (i, j = 1, 2, \dots, n) \tag{12}$$

$$M = (m_{ij})_{n \times n} \tag{13}$$

The weights of each evaluation index can be calculated based on the judgment matrix M . The specific solution process is referred to the references [30] and [31]. The subjective weight vector W^s is expressed as:

$$W^s = (w_1^s, w_2^s, \dots, w_n^s) \tag{14}$$

Where w_i^s ($i = 1, 2, \dots, n$) is the weight value of the i th evaluation index. Then, the consistency test of the initial weight of the evaluation index is carried out, and the specific calculation steps are referred to the references [30] and [31].

Objective Weighting Method

According to the statistical analysis of more than 100 cases of water and mud inrush in karst tunnels, the relationships between each influencing factor and the occurrence of water inrush were determined by the frequency statistics method [4]. The objective weight of each evaluation index was obtained as $W^o = [0.155, 0.349, 0.173, 0.095, 0.039, 0.130, 0.058]$.

Grading Criteria of Risk Evaluation Index

Based on the existing research results [4, 7, 31], seven factors including formation lithology I_1 , unfavorable geology I_2 , groundwater level I_3 , topography and geomorphology I_4 , the attitude of rock formation I_5 , contact zone of dissolvable and insoluble rocks I_6 , and layer and interlayer fissures I_7 are selected as the risk assessment indices of water inrush. According to the risk degree of water inrush, the C_1 (very high risk), C_2 (high risk), C_3 (medium risk), and C_4 (low risk) from high to low are divided. The detailed grading criteria are shown in Table 3.

Results and Discussion

Engineering Verification

The Xiakou tunnel is a high-risk karst tunnel of the Yiba Expressway. There is a great possibility of karst

Table 2. Triangular fuzzy scaling method for evaluation index.

Scale	Triangular fuzzy number	Linguistic scale for importance
1%	(1/2, 1, 3/2)	I_i and I_j are equally important
3%	(5/2, 3, 7/2)	I_i is slightly more important than I_j
5%	(9/2, 5, 11/2)	I_i is obviously more important than I_j
7%	(13/2, 7, 15/2)	I_i is strongly more important than I_j
9%	(17/2, 9, 19/2)	I_i is absolutely more important than I_j
2%, 4%, 6%, 8%	(3/2, 2, 5/2), (7/2, 4, 9/2), (11/2, 6, 13/2), (15/2, 8, 17/2)	Middle value of upper and lower scales respectively

development in the strata passing through or near the tunnel, especially the Triassic Daye Formation (T_{1d}) and Jialingjiang Formation (T_{1-2j}) [8, 9]. Therefore, the inclined shaft of the Xiakou tunnel is selected, and the proposed method is used to evaluate the risk level of water inrush for the inclined shaft section XJK0+110 ~ XJK0+060. The value intervals of the evaluation indices are shown in Table 4 [8, 9].

Integrated Weighting Determination

The established TFN-AHP is used to calculate the subjective weights of the evaluation indices of tunnel water inrush. Firstly, the judgment matrix is constructed based on the scaling method. Then, the subjective weights of the indices are obtained by Eq. (12) ~ Eq. (14). Finally, the consistency test of the judgment matrix is carried out by Equations included in reference [31]. It can be seen from Table 5 that the matrix satisfied the consistency test.

The subjective and objective weight distribution coefficients are selected to be 0.5. Combined with the above objective weights, the integrated weights are derived as follows:

$$W = [0.159, 0.308, 0.220, 0.096, 0.048, 0.113, 0.058] \tag{15}$$

Risk Attribute Identification of Water Inrush

According to the grading criteria of the evaluation indices in Table 3, the parameters in Eq. (1) to Eq. (6) are quantified, and the single-index attribute measure functions are constructed, as shown in Fig. 1. Among them, the grading method of the evaluation indices $I_1, I_2, I_5, I_6,$ and I_7 is based on the expert scoring. Therefore, their single-index attribute measure functions are the same. By substituting the value intervals of the evaluation indices in Table 4 into the functions in Fig.1, the single attribute measure values are obtained. According to Eq. (7) ~ (8), the synthetic attribute measure values are calculated, as shown in Table 6.

Eq. (9) is selected to calculate the risk level of water inrush, where the confidence level λ is taken as 0.65. The risk level of the section XJK0+110 ~ XJK0+060 from the inclined shaft of the Xiakou tunnel is C_1 . Different methods have been used to evaluate the risk level of water inrush in this case before, and the results are C_1 [8, 9]. During actual excavation, many serious water inrush occurred in the section XJK0+110 ~ XJK0+060 [8].

Table 3. Evaluation indices and grading criteria of water inrush [4-6].

Index	C_1	C_2	C_3	C_4
Formation lithology I_1	Strongly soluble rock	Medium soluble rock	Weakly soluble rock	Non-soluble rock
	85~100	70~85	60~70	0~60
unfavorable geology I_2	Large water-bearing and water-conducting structure	Medium-sized water-bearing and water-conducting structure	Small water-bearing and water-conducting structure	No water-bearing and water-conducting structure
	85~100	70~85	60~70	0~60
Groundwater level I_3/m	60~90	30~60	10~30	0~10
Topography and geomorphology I_4	Large negative landform	Medium negative landform	Small negative landform	No negative landform
	>60%	40%~60%	20%~40%	0~20%
Attitude of rock formation I_5°	25°~45° or 45°~65°	10°~25° or 65°~80°	80~90°	0~10°
	85~100	70~85	60~70	0~60
Contact zone of dissolvable and insoluble rocks I_6	Strongly conducive to karst development	Moderately conducive to karst development	Weakly conducive to karst development	Slightly conducive to karst development
	85~100	70~85	60~70	0~60
Layer and interlayer fissures I_7	Strongly conducive to karst development	Moderately conducive to karst development	Weakly conducive to karst development	Slightly conducive to karst development
	85~100	70~85	60~70	0~60

Table 4. Value intervals of evaluation indices for XJK0+110 ~ XJK0+060.

Index	I_1	I_2	I_3	I_4	I_5	I_6	I_7
$[t_{ix}, t_{ij}]$	[75, 80]	[90, 95]	>75	[25, 30]	40	[70, 75]	[85, 90]

Table 5. Judgment matrix for subjective weights analysis.

Index	I_1	I_2	I_3	I_4	I_5	I_6	I_7
I_1	(1/2, 1, 3/2)	(2/5, 1/2, 2/3)	(2/5, 1/2, 2/3)	(3/2, 2, 5/2)	(5/2, 3, 7/2)	(3/2, 2, 5/2)	(5/2, 3, 7/2)
I_2	(3/2, 2, 5/2)	(1/2, 1, 3/2)	(1/2, 1, 3/2)	(5/2, 3, 7/2)	(7/2, 4, 9/2)	(5/2, 3, 7/2)	(7/2, 4, 9/2)
I_3	(3/2, 2, 5/2)	(1/2, 1, 3/2)	(1/2, 1, 3/2)	(5/2, 3, 7/2)	(7/2, 4, 9/2)	(5/2, 3, 7/2)	(7/2, 4, 9/2)
I_4	(2/5, 1/2, 2/3)	(2/7, 1/3, 2/5)	(2/7, 1/3, 2/5)	(1/2, 1, 3/2)	(3/2, 2, 5/2)	(1/2, 1, 3/2)	(3/2, 2, 5/2)
I_5	(2/7, 1/3, 2/5)	(2/9, 1/4, 2/7)	(2/9, 1/4, 2/7)	(2/5, 1/2, 2/3)	(1/2, 1, 3/2)	(2/5, 1/2, 2/3)	(1/2, 1, 3/2)
I_6	(2/5, 1/2, 2/3)	(2/7, 1/3, 2/5)	(2/7, 1/3, 2/5)	(1/2, 1, 3/2)	(3/2, 2, 5/2)	(1/2, 1, 3/2)	(3/2, 2, 5/2)
I_7	(2/7, 1/3, 2/5)	(2/9, 1/4, 2/7)	(2/9, 1/4, 2/7)	(2/5, 1/2, 2/3)	(1/2, 1, 3/2)	(2/5, 1/2, 2/3)	(1/2, 1, 3/2)
Weight	0.163	0.266	0.266	0.096	0.057	0.096	0.057
Consistency test	$\lambda_{max} = 7.103, CI = 0.017, CI = 0.013$, the judgment matrix satisfies the consistency test						

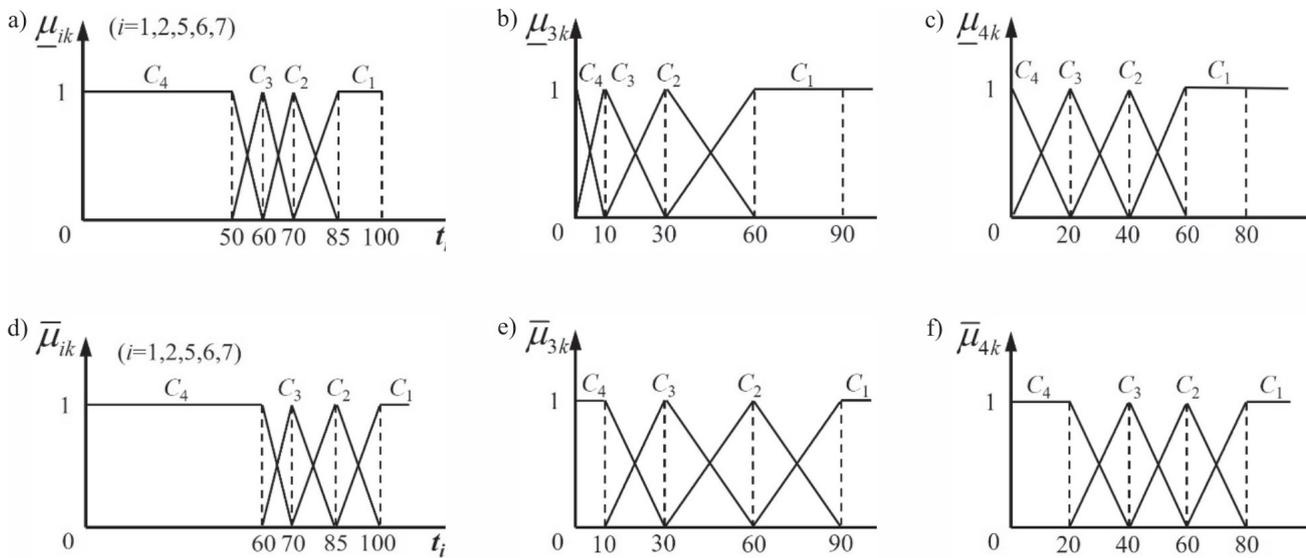


Fig. 1. Single-index attribute measure functions of the evaluation indices for water inrush risk assessment.

Table 6. Single-index and synthetic Attribute measure values of the section XJK0+110 ~ XJK0+060.

Attribute measure		C_1	C_2	C_3	C_4
μ_{ik}	I_1	0.250	0.500	0.250	0
	I_2	0.750	0.250	0	0
	I_3	0.875	0.125	0	0
	I_4	0	0.188	0.500	0.313
	I_5	0.875	0.125	0	0
	I_6	0.083	0.500	0.417	0
	I_7	0.583	0.417	0	0
μ_k		0.548	0.289	0.135	0.030

It is verified that the fuzzy attribute interval assessment method proposed in this paper is accurate and reliable.

Engineering Application

Yunwushan Tunnel is a key control project of the Chongqing suburban railway from Bishan to Tongliang line, which is a single-bore double-line tunnel. The mileage is from YDK14+025 to YDK19+835 with a total length of 5810 m and a maximum buried depth of 343 m. The tunnel passes through the core of Libixia anticline, F1 fault, F2 fault, F3 fault, and Longjiacao reverse fault in turn, the geological conditions are very complicated. The groundwater in the tunnel area is rich with loose pore water and bedrock fissure water

dominating in the sandstone and mudstone sections, and karst water dominating in karst development sections. The adverse geology such as coal mine goaf, karst, and fault in tunnel area is developed, the risks of water and mud inrush, and large volume collapse are high.

The proposed attribute fuzzy interval assessment method of water inrush is applied to evaluate the water inrush risk of the concealed-digging section YDK14+030~YDK19+835 of Yunwushan Tunnel. According to the geological survey and special hydrological survey data of the tunnel, the measured interval values of the evaluation indices are determined, as shown in Table 7.

Table 7. Measured value intervals of evaluation indices for Yunwushan Tunnel.

Mileage	I_1	I_2	I_3/m	$I_4/\%$	I_5	I_6	I_7
YDK14+030~+185	Mudstone and shale intercalated with sandstone	Karst structures are not developed	Undeveloped	The Baisha river is developed nearby	The dip angle of rock stratum is 70~80°	Undeveloped	Weakly developed
	[55, 60]	[50, 55]	[20, 24]	[30, 40]	[70, 75]	[50, 55]	[60, 65]
YDK14+185~+440	Thick sandstone intercalated with shale, which is moderately permeable	There are low resistance anomaly areas. Karst structures are weakly developed	Relatively developed	Weakly developed	The dip angle of rock stratum is 73~80°	Mudstone and sandstone mutation	Undeveloped
	[70, 75]	[60, 65]	[60, 64]	[20, 30]	[70, 75]	[60, 65]	[50, 55]
YDK14+440~+490	Medium-thick shale intercalated with coal line	Karst structures are not developed	Relatively developed	Undeveloped	The dip angle of rock stratum is 73~80°	Undeveloped	Weakly developed
	[50, 55]	[55, 60]	[78, 80]	[15, 20]	[70, 75]	[50, 55]	[60, 65]
YDK14+490~+635	Thick sandstone, which is moderately permeable	Karst structures are weakly developed	Relatively developed	Weakly developed	The dip angle of rock stratum is 73~80°	Undeveloped	Weakly developed
	[70, 75]	[60, 65]	[85, 89]	[20, 30]	[70, 75]	[50, 55]	[60, 65]
YDK14+635~+835	Medium-thick shale intercalated with sandstone and coal seam	The mined-out area is located below 45m of the tunnel floor	Developed	Weakly developed	The dip angle of rock stratum is 72~80°	Sandstone and shale mutation	Weakly developed
	[50, 55]	[60, 65]	[102, 112]	[20, 30]	[70, 75]	[65, 70]	[60, 65]
YDK14+835~YDK15+010	Dolomitic limestone, dolomite, breccia	There are low resistance anomaly areas. Karst structures are developed	Developed	Developed	The dip angle of rock stratum is 72~80°	Shale and limestone mutation	Moderately developed
	[80, 85]	[80, 85]	[99, 110]	[50, 60]	[70, 75]	[85, 90]	[70, 75]
YDK15+010~YDK15+530	Medium-thick limestone	There is Libixia anticline. Karst structures are developed	Relatively developed	Developed	The dip angles are 80~84° (west wing) and 10°~28° (east wing)	Breccia and limestone mutation	Moderately developed
	[85, 90]	[85, 90]	[72, 77]	[50, 60]	[75, 80]	[75, 80]	[70, 75]

Table 7. Continued.

YDK15+530~+880	Dolomitic limestone, dolomite, breccia	There are medium-low resistance anomaly areas, the karst structures are relatively developed	Relatively developed	Developed	The dip angle of rock stratum is 60~78°	Limestone-muddy limestone-shale mutation in turn	Moderately developed
	[80, 85]	[75, 80]	[71, 72]	[50, 60]	[75, 80]	[80, 85]	[70, 75]
YDK15+880~YDK16+130	Shale intercalated with sandstone and coal seams	Karst structures are not developed	Very developed	Weak development of negative landform	The dip angle of rock stratum is 60~78°	Undeveloped	Weakly developed
	[50, 55]	[50, 55]	[150, 152]	[20, 30]	[70, 75]	[50, 55]	[60, 65]
YDK16+130~+460	Dolomitic limestone, dolomite, breccia	There is F2 reverse fault. Karst structures are developed	Very developed	Developed	The dip angle of rock stratum is 20°~28°	Limestone and shale Mutation	Moderately developed
	[80, 85]	[80, 85]	[110, 145]	[50, 60]	[80, 90]	[85, 90]	[70, 75]
YDK16+460~+760	Medium-thick shale intercalated with sandstone and coal seams	Karst structures are not developed	Very developed	Weak development of negative landform	The dip angle of rock stratum is 22°	Undeveloped	Weakly developed
	[50, 55]	[50, 55]	[150, 151]	[20, 30]	[70, 75]	[50, 55]	[60, 65]
YDK16+760~YDK17+110	Thick sandstone intercalated with shale, which is moderately permeable	Karst structures are weakly developed	Developed	Weak development of negative landform	The dip angle of rock stratum is 22°	Sandstone and shale Mutation	Weakly developed
	[70, 75]	[60, 65]	[102, 125]	[30, 40]	[75, 80]	[65, 70]	[60, 65]
YDK17+110~+610	Sandstone intercalated with shale, which is moderately permeable	There is F1 reverse fault	Relatively developed	Weak development of negative landform	The dip angle of rock stratum is slow	Sandstone and shale Mutation	Weakly developed
	[70, 75]	[70, 75]	[68, 69]	[30, 40]	[75, 80]	[65, 70]	[60, 65]
YDK17+610~YDK18+070	Thick sandstone intercalated with shale, which is moderately permeable	Karst structures are weakly developed	Relatively developed	There is large negative landform development	The dip angle of rock stratum is 28°	Sandstone and shale Mutation	Moderately developed
	[75, 80]	[60, 65]	[70, 75]	[65, 70]	[85, 90]	[65, 70]	[70, 75]
YDK18+070~YDK19+185	Mudstone and sandstone intercalated with shale	There is Longjiacao fault	Relatively developed	There is Sanjiang Reservoir	The dip angle of rock stratum is 28°	Sandstone and Shale Mutation	Moderately developed
	[75, 80]	[70, 75]	[62, 69]	[70, 75]	[85, 90]	[70, 75]	[70, 75]
YDK19+185~+835	Mudstone and shale intercalated with sandstone	Karst structures are not developed	Weakly developed	Weak development of negative topography	The dip angle of rock stratum is 28°	Undeveloped	Weakly developed
	[55, 60]	[55, 60]	[20, 25]	[30, 40]	[75, 80]	[50, 55]	[60, 65]

Table 8. Synthetic attribute measure matrix of water inrush risk in Yunwushan Tunnel.

Mileage	Synthetic attribute measure (μ_k)				Risk level
	C_1	C_2	C_3	C_4	
YDK14+030 ~ +185	0.004	0.319	0.133	0.546	C_3
YDK14+185 ~ +440	0.135	0.271	0.352	0.245	C_2
YDK14+440 ~ +490	0.184	0.072	0.241	0.506	C_3
YDK14+490 ~ +635	0.171	0.227	0.331	0.272	C_2
YDK14+635 ~ +835	0.224	0.124	0.327	0.327	C_3
YDK14+835 ~ YDK15+010	0.525	0.382	0.095	0	C_1
YDK15+010 ~ YDK15+530	0.517	0.409	0.076	0	C_1
YDK15+530 ~ +880	0.395	0.459	0.148	0	C_1
YDK15+880 ~ YDK16+130	0.224	0.043	0.170	0.565	C_3
YDK16+130 ~ +460	0.545	0.378	0.079	0	C_1
YDK16+460 ~ +760	0.224	0.043	0.170	0.565	C_3
YDK16+760 ~ YDK17+110	0.245	0.220	0.366	0.171	C_2
YDK17+110 ~ +610	0.192	0.415	0.340	0.055	C_2
YDK17+610 ~ YDK18+070	0.281	0.277	0.274	0.130	C_2
YDK18+070 ~ YDK19+185	0.313	0.410	0.240	0.037	C_2
YDK19+185 ~ +835	0.012	0.136	0.388	0.466	C_3

Risk Assessment of Water Inrush

The measured interval values of the evaluation indices in Table 7 are substituted in the functions in Fig. 1. The synthetic attribute measure values for the water inrush risk of each section in the Yunwushan Tunnel are obtained, as shown in Table 8. Based on Eq. (9) with the confidence level $\lambda = 0.65$, the water inrush risk level of each section of the Yunwushan Tunnel can be obtained, as shown in Table 8.

Excavation Situation

When the tunnel is constructed to the working face YDK15+316, water inrush occurs during the blast-hole drilling. The jet distance along the blast-hole is about 10m, and the hydrodynamic pressure is about 0.1 MPa. The location of the water outlet point is 1.4 m away from the bottom of the upper step, as shown in Fig. 2. It can be seen from Table 6 that the risk level of water inrush in this section is C_2 (High risk), which verifies that the assessment results are in good agreement with the actual situation.



Fig. 2. Water inrush situation of the tunnel face.

Conclusions

(1) Aiming at the complexity of geological conditions and the uncertainty of the evaluation index values, a fuzzy attribute interval assessment model of tunnel water inrush is proposed based on the attribute mathematical theory. The evaluation index is quantified by using a small-range mathematical interval, and then the upper and lower limit probability functions of single-index attribute measure are constructed, and the calculation method of the synthetic attribute measure is presented.

(2) To improve the accuracy of the evaluation index weights, a combination weighting method combining the subjective weights and the objective weights is adopted. The triangular fuzzy number theory is introduced

to improve the AHP, and a new TFN-AHP is established to calculate the subjective weights of the evaluation indices, which can overcome the uncertainty and hesitation of experts in judging the relative importance of the evaluation indices.

(3) The evaluation index system of tunnel water inrush and its grading criteria are put forward. The proposed fuzzy attribute interval assessment method of water inrush risk is applied to the concealed-digging section YDK14+030~YDK19+835 of the Yunwushan Tunnel. By comparing the assessment results with the actual situation, the rationality and feasibility of the method are verified.

The proposed method provides a way for the risk recognition of tunnel water inrush. However, the assessment process involves some subjectivity, including the quantitative value of some indices and the determination of subjective weights for evaluation indices. The directly measured assignment method and calculation method of accurate weight are worth exploring in the future.

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Conflict of Interest

The authors declare no conflict of interest.

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